6.2 Gear pair

6.2.1 External gear pair

Gear pair with parallel axes

Gear pair with parallel axes is composed by two matching cylindrical gears. Gear pair with parallel axes is a kind of face gear pair, which has the function of transmitting the operation and torque of two parallel axes. It can be classified into several kinds, see Figure 6-23.

Cylindrical gear pair

Cylindrical gear pair is composed of two matched cylindrical gears (Figure 6-23). It can be classified into two categories: cylindrical gear with parallel axes and cylindrical gear with alternating shaft. Generally, cylindrical gear with parallel axes is called cylindrical gear.

The function of cylindrical gear is transmitting the operation and torque of two parallel axes and alternating shaft.

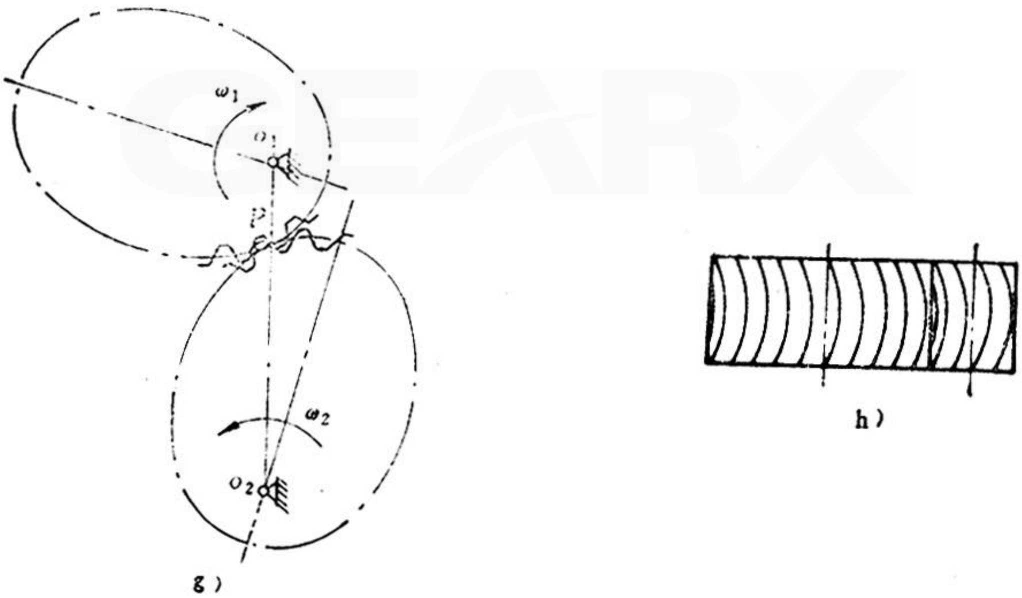


图 6-23

a）外啮合直齿圆柱齿轮副 b）内啮合直齿圆柱齿轮副 c）斜齿圆柱齿轮副
d）人字齿圆柱齿轮副 e）齿条副 f）圆弧圆柱齿轮副 g）椭圆齿轮副
h）曲齿线圆柱齿轮副

External gear pair

External gear pair is a kind of conjugate gear composed of two external gears. External gear pair is the most commonly seen among various gears.

Spur gear pair

Spur gear pair is gear with parallel axes composed of two matched spur gears. It is commonly refers to involute spur gear, shortly, spur gear (Figure 6-24).

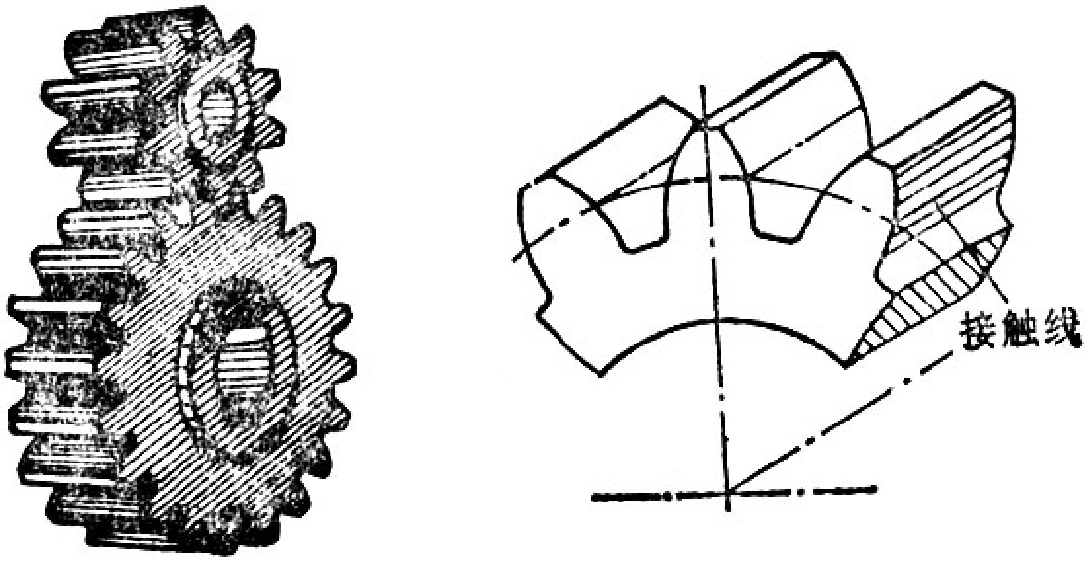


图 6-24

The function of spur gear is transmitting the operation and torque of two parallel axes. It is the fundamental form of gear pair with convenient design and good performance. The instantaneous contact line is a straight line that is parallel to the shaft line. It contacts along the whole teeth length and is easily bringing forth impact, vibration and offset loading. It has less teeth number and small contact ratio. It is mainly used in small and medium-size modulus gear with small loading and slow speed or shifting gear pair installed on machine.

Involute gear pair

Gear pair composed of matched gear with involute tooth profile is often called involute gear pair. It is a generalized term. And it is used to be called involute cylindrical gear.

Involute gear pair with parallel axes

This kind of gear pair consists of two matched involute cylindrical gear. It can be classified into three types: rack-and-pinion, external gearing and internal gearing gear pair. The tooth orientation can be classified straight tooth, helical tooth and herringbone tooth. The characteristics of involute cylindrical gear with parallel shaft are as follows: transverse path of contact is the common tangent working on the base circle; contact surface is the common surface tangent to the base cylinder; during the engagement process and at the engagement point, the common normal and line of centers of tooth profile are intersecting on the fixed point P, that means the instant center remains unchanged, and there will be a constant transmission ratio; during the engagement process, the normal force $F_n\left(F_n=\frac{T}{r_b}\right)$ beard by tooth surface remain the same, so does the direction; the center distance change slightly while the instant transmission ratio remain unchanged, that means the center distance is separable; in the transverse plane, common tangent line, common normal line, meshing line, involute generating line and the application line of force are coincident; the transmitting quality can be improved through modification and profiling; gear can be manufactured with indirect generating method; the range of transmission speed, capacity and gear ratio is large with high efficiency, various types and wide application; the requirements on processing, installation and repair is quite strict. The cost is relatively high and the work noise is relatively big.

Involute spur gear pair of external gearing with parallel axes

It is a kind of gear pair with parallel axes composed of two matched involute spur gear of external gearing. The gearing characteristics can be seen from “involute cylindrical gear with parallel axes” and “spur gear pair”.

Gear pair

Gear pair is consisting of two conjugate meshing gears. It is the simplest form of combination for gears to transmit operation and torque.

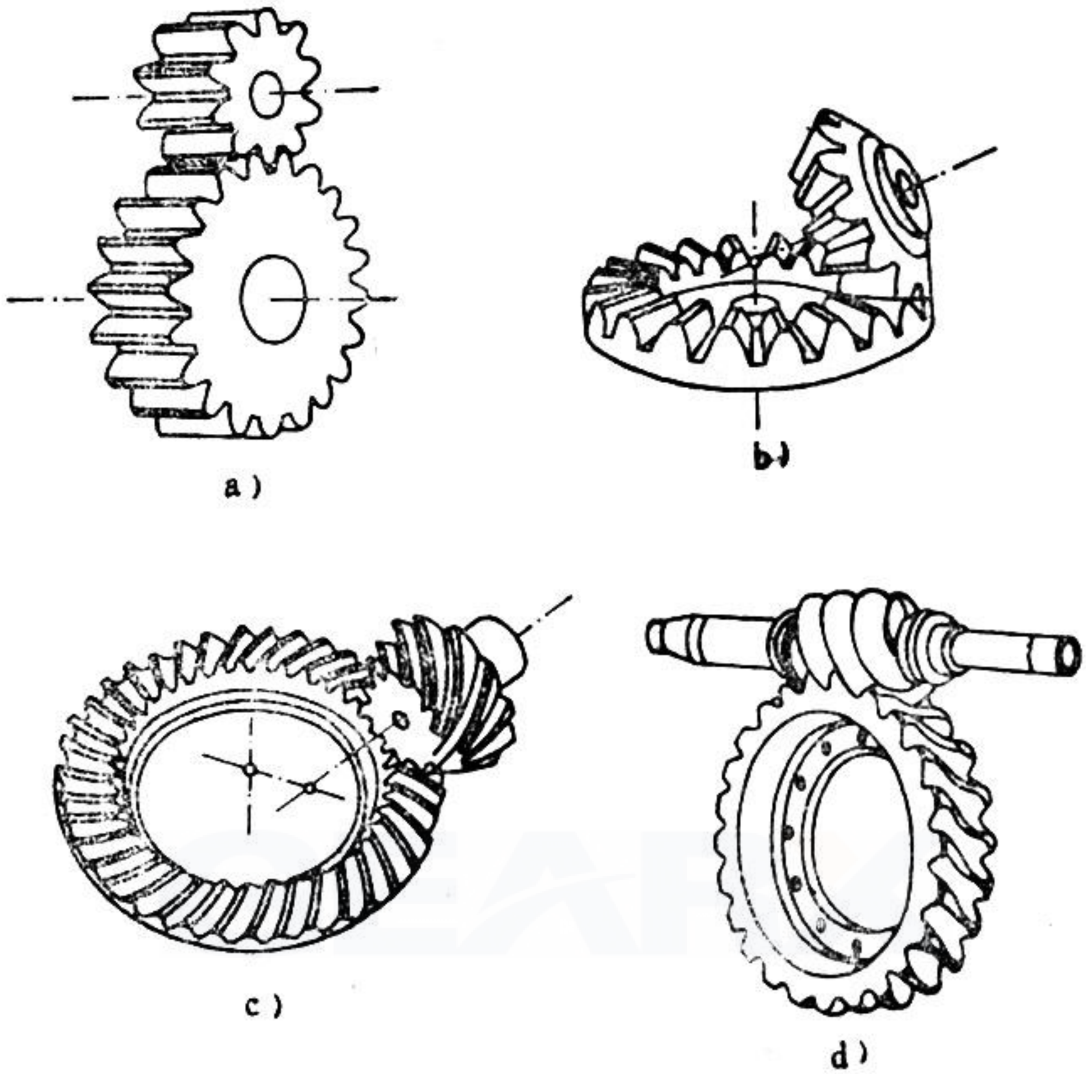


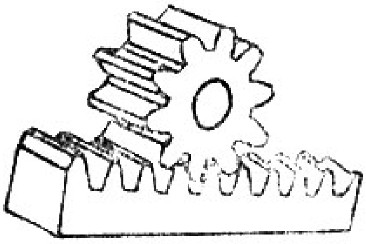
图 6-25

Gear drive

Gear pair or gear train used to transmit operation and rotational moment is collectively called gear drive. During the meshing process, a pair of conjugate gears can be called gear pair, which can also be called gear drive. Gear drive is a common term used to replace gear pair.

Rack pair

Gear pair (Figure 6-26) consists of matched gear and rack is called rack pair. It is a kind of gear mechanism that can shift rotation (or movement) into movement (or rotation). It can be classified into spur gear pair (Figure 6-26a) and helical gear pair (Figure 6-26b). The most commonly seen form is spur gear pair with straight profile. It is a special form of straight tooth involute cylindrical gear pair. Its meshing characteristic is that the reference circle and pitch circle of gear are overlapped ($r_1 \equiv r_1'$); the meshing angle and pressure angle are equal ($a \equiv a'$); the circumferential speed of gear is equal to the traveling speed of rack. With the engagement principle of rack pair, involute gear can be manufactured under generating method taking rack as cutting tool. Both direct generating method and indirect generating method have good performance.



a)

b)

图 6-26

a) 直齿条副 b) 斜齿条副

Spur rack pair

It refers to gear pair composed of spur gear and its matched cylindrical spur gear. It is a simple and convenient gear mechanism that can turn rotation into movement. It can also turn movement into rotation, but it is rarely seen.

Helical rack pair

Helical rack pair is composed of helical rack and its matched helical gear. The function of helical rack pair is the same to that of spur gear pair. But the management character is superior to the latter, so does the bearing capacity.

Standard gear pair

Standard gear pair is composed of two matched standard gear pairs. The characters can be seen from “standard gear pair”

Standard gear pair

Standard gear pair is composed by two standard gears. The main features are: $a = a$, $r' = r$, $a = a'$, $y = 0$ .Stand gear pair has good performance in both workmanship and interchangeability.

Although standard gear pair has many advantages, there are lots of defects. For example, it is hard for standard gear pair to meet the requirement of the following: to avoid undercutting and make the structure of gear pair more compact; to satisfy the requirement on center distance and transmission ratio; to improve transmission efficiency; to try to match the equal strength of meshing gear to improve the bearing capacity, etc.

Helical gear pair

Helical gear pair is the abbreviation of “helical cylindrical gear pair”. But generally, it refers to helical gear pair with parallel axes.

Involute helical gear pair of external gearing with parallel axes

Involute helical gear pair of external gearing with parallel axes consists of two matched involute helical gear of external gear. Detail please sees “helical gear pair with parallel axis” and “involute cylindrical gear pair with parallel axes”.

Helical gear pair

In gear pair, helical gear pair refers to gear pair with at least one helical gear. It can be mainly classified into two categories: helical gear pair with parallel axes (Figure 6-27a) and helical gear pair with alternating shaft (Figure 6-27b). Generally, it refers to the former.

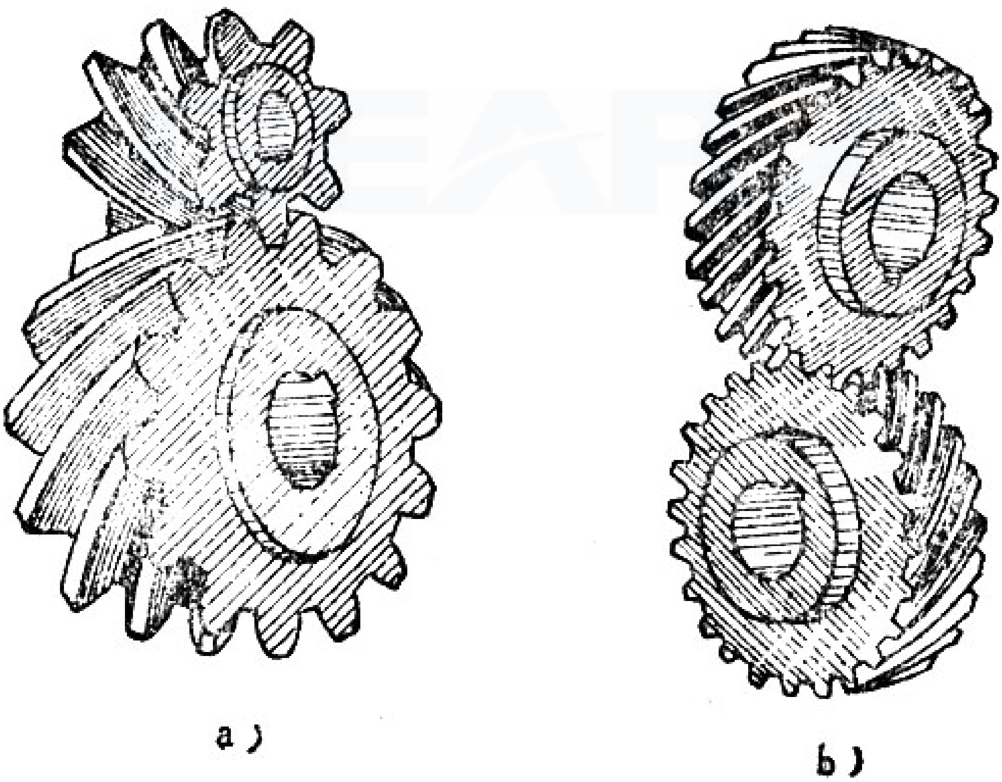


图 6-27

Parallel helical gear pair

Parallel helical gear is composed of two matched helical gears (Figure 6-28a). In general, it refers to involute helical gear pair with parallel axes. It is used to transmit the operation and turning moment of the two parallel axes.

Compared to spur gear pair with parallel axes, the main features of helical gear pair with parallel axes are (taking involute gears as example): β_b angle is formed between the instant contact line and bus bar of the base cylinder (Figure 6-28b); the length of shortest contact line is longer ($L_{min} = \varepsilon_a b / \cos\beta_b$); it has large overlap size, stable working performance, compact structure and large overcapacity; center distance can be managed by adjusting the angle β; it can be used in heavy load with high speed occasion; large axial force; it is unsuited to be used in slide modified gear.

With regard to helical gear pair with parallel axes, the most commonly used are involute helical gear pair of external gearing with parallel axes and involute helical gear pair of internal gearing with parallel axes.

From the Figure 6-28b, we can see the contact line on the gear teeth of involute helical gear pair of external gearing with parallel axes.

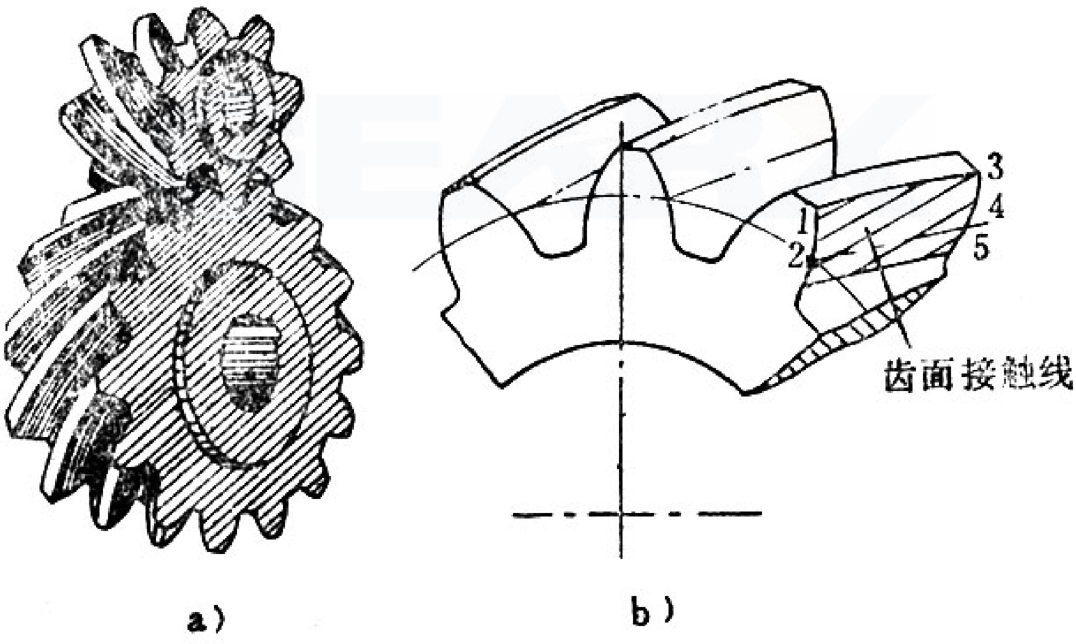


图 6-28

Virtual gear pair of helical gear, virtual gear pair

The imaginary spur gear pair (Figure 6-29) composed of two virtual gears of helical gear in helical gear pair is called the virtual gear pair of helical gear pair. It serves as the fundamental basis in researching the meshing characteristics, law of motion and strength calculation of helical gear pair.

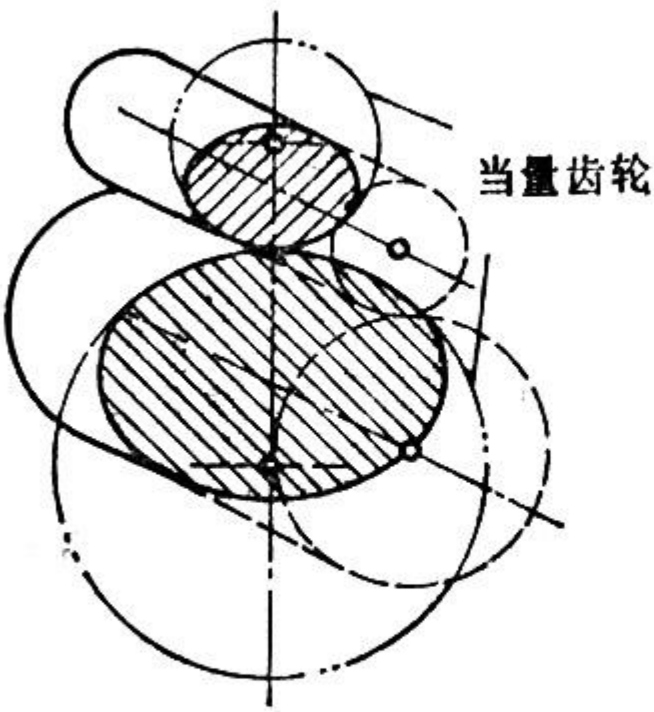


图 6-29

Characteristics of gear drive

Compared with other machinery drive like belts and chains, gear drive has many other characteristics: it can realize instantaneous constant, or average constant or with vested variation laws' transmission ratio; with large change range of transmission ratio, it can constitute speed reducing or increasing mechanism; it has high transmission efficiency; it has large dimensional range, various kinds and wide range of application fields; it has high summation watt rating and compact structure; it requires high precision, with complicated workmanship and high cost; it has no overloading protection, so infinitely variable speeds cannot be realized, and there are big noises.

Herringbone gear pair

Herringbone gear pair is composed of two matched herringbone gears. It can be classified into two types: external herringbone gear pair and internal herringbone gear pair.

Involute herringbone gear pair of external gearing with parallel axes

It consists of two matched involute herringbone gears of external gearing. Details can be seen from "involute gear pair with parallel axes" and "herringbone gear pair of external gearing".

Herringbone gear pair of external gearing

Herringbone gear pair of external gearing is composed of two matched herringbone gears of external gearing. In addition to all the characteristics of helical gear pair of external gearing, it can also balance the axial force autonomously during the meshing process. Its gearing features are superior to helical gear pair because of its larger helix angle. But it has larger axial dimension.

X-gear pair，modified gear pair

X-gear pair is the general name of “radial modified gear pair” and “tangential modified gear pair”. Generally, however, it refers to the former.

Gear pair of addendum modification，X-gear pair

It refers to gear pair which has at least one radial modified gear. And in short, it is called X-gear pair. It can be classified as follows (Figure 6-30);

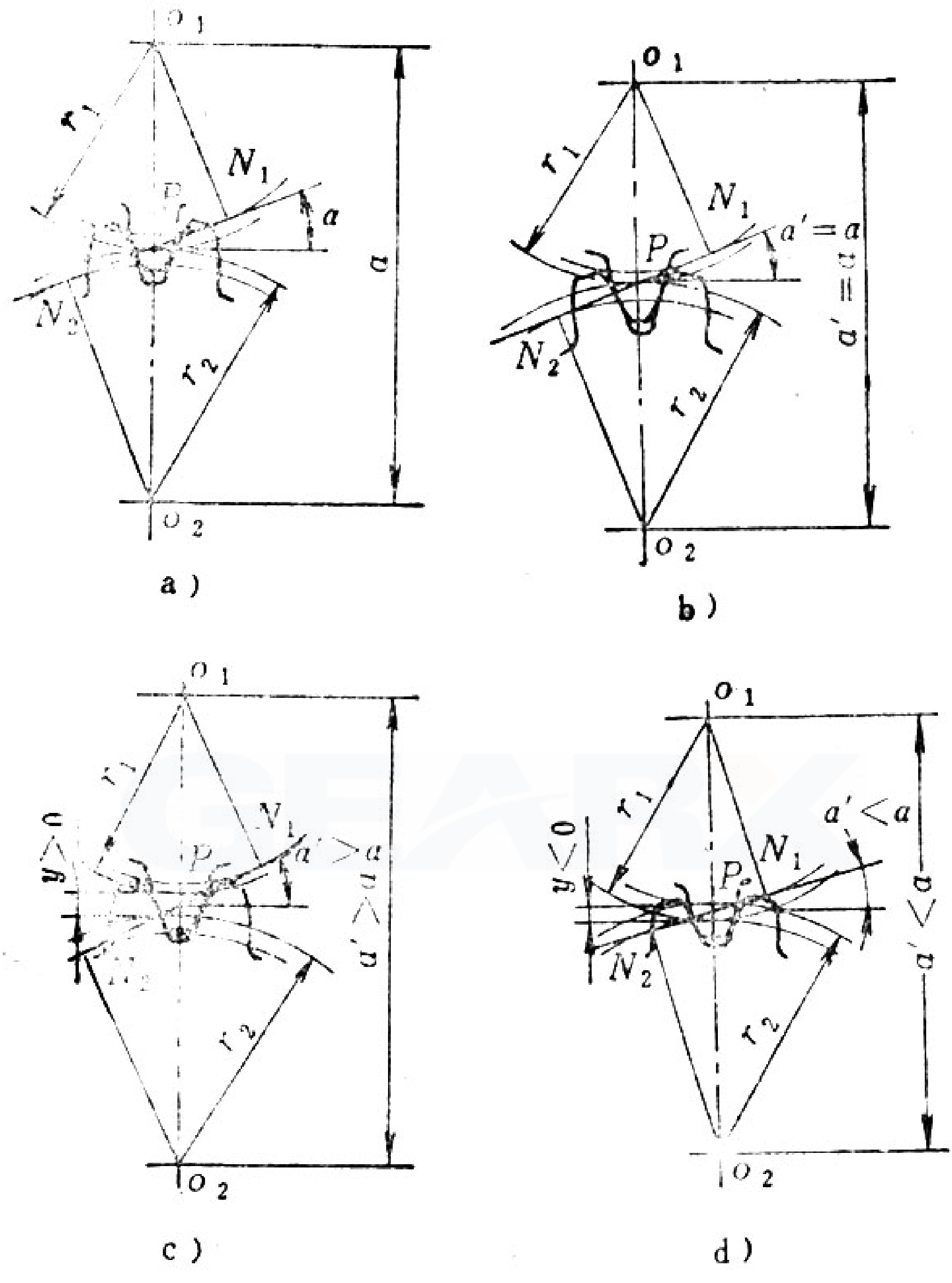


图 6-30

a）标准齿轮副 b）高变位齿轮副 c）正角变位齿轮副 d）负角变位齿轮副

V-gear pair

V-gear pair is the code name of angle modified gear pair stipulated in Germany DIN870 (1931).

V-O gear pair

V-O gear pair is the code name of addendum modification gear pair stipulated in Germany DIN870 (1931).

Gear pair with modified center distance

Cylindrical gear pair with at least one modified gear and the sum (x_1+x_2) of modification coefficient of two cylindrical gears are not zero, then this kind of cylindrical gear pair is called gear pair with modified center distance.

Compared with standard gear pair, the engagement parameter of gear pair with modified center distance has a lot of differences: the nominal center distance is not equal to standard center distance ($a' \neq a$), so does the relation between engagement angle and pressure angle ($a' \neq a$), and the pitch circle and reference circle are not overlapped; both the center distance change (my) and addendum circle change (Δym) are not zero.

Gear pair with modified center distance can be divided into two types: it is called positive gear pair with modified center distance when the nominal center distance is larger than the standard center distance ($a'>a$) ; it is called negative gear pair with modified center distance when the nominal center distance is smaller than the standard center distance ($a'<a$).

Cylindrical gear pair with positive modified center distance

When the sum of modification coefficient of two matched gears is larger than zero, then this kind of gear pair with modified center distance is called positive gear pair with modified center distance. Compared with standard cylindrical gear pair, the engagement parameter of cylindrical gear with positive modified center distance changes as follows: the nominal center distance is larger than the standard center distance ($a'>a$) and the diameter of pitch circle is larger than the diameter of reference circle ($d_1'>d_1$, $d_2'>d_2$); the engagement angle is larger than the pressure angle ($a'>a$); the modification coefficient of center distance is larger than zero ($y>0$); the modification coefficient of addendum circle (Δy) is not zero, etc.

Compared with standard cylindrical gear, cylindrical gear pair with positive modified center distance has features like: it can reduce the size of gear mechanism and mitigate tooth surface abrasion, and both the bending strength and tooth surface contacting strength of gear teeth can be relatively improved; it has flexible design and can meet the special satisfaction of center distance and speed ratio; it must be designed in pairs and the design, manufacture, application and the interchangeability is relatively poor; the overlap ratio reduced slightly.

In short, cylindrical gear pair with positive modified center distance is a good driving form with a range of merits and wide usage.

Gear pair with negative modified center distance

Cylindrical gear pair whose sum of modification coefficient of two matched gears is smaller than zero is called cylindrical gear pair with negative modified center distance. Compared with standard cylindrical gear pair, its engagement parameter has a lot of changes: $a'<a$, $r_1'<r_1$, $r_2'<r_2$, $a'<a$, $y<0$, $\Delta y\neq 0$.

Compared with standard cylindrical gear pair, gear pair with negative modified center distance has characteristics as follows: relatively large overlap ratio; it can meet the requirement on design; it has poor interchangeability because it has to be designed, manufactured and applied in pairs; the largest sliding coefficient of dedendum is increased with larger gear teeth abrasion; both the bending strength and contact strength of gear teeth decrease. In short, gear pair with negative modified center distance has relatively more defects, so it only be applied in unavoidable condition or used to make up the center distance.

Gear pair with reference center distance

Gear pair composed of a pair of modified gears whose modification coefficient is $x_1=-x_2\neq 0$ is called gear pair with reference center distance. The engagement characteristics of this kind of gear are: the nominal center distance is equal to standard center distance ($a'=a$); the meshing angle is equal to pressure angle ($a'=a$); the pitch circle is overlapped with reference circle ($r_1'=r_1$、$r_2'=r_2$); the center distance modification coefficient y=0, the addendum circle modification circle Δy=0.

Compared to standard cylindrical gear pair, gear pair with reference center distance has the following features: the strength of gear pair can be improved without changing the size of box and structure; z_1 can be smaller than z_{min}, so the structure of the gear drive is compact; it has poor interchangeability because it has to be designed, manufactured and applied in pairs; it must meet the requirement of $z_1+z_2\geq 2z_{min}$; it is not easy to adjust the center distance and the overlap ratio is smaller.

Combination modified gear pair

This kind of gear pair are consist of two matched combination modified gears. Most of these kinds of modified gear pair are bevel gear pair, such as modified bevel gear pair with reference-tangent center distance.

Tangent modified gear pair

Tangent modified gear pair is composed of tangent modified gear.

Gear pair with meshing out circle pitch point

During the engagement process of cylindrical gear pair, the significant tooth surface of a gear is located between the pitch cylinder and addendum cylinder, the significant tooth surface of the other gear is located between the pitch cylinder and dedendum cylinder. This means gear with meshing out circle pitch point has non-intersect cylindrical gear pair. It can be composed of two matched gears with big modification; it can also be designed directly (such as circular arc gear pair). Gear pair with meshing out circle pitch point has good meshing performance.

Gear pair with meshing ahead pitch point

In cylindrical gear pair, the significant tooth surface of driving gear is located between the pitch cylinder and dedendum cylinder, while the significant tooth surface of driven gear is located between the pitch circle and addendum cylinder. Then this kind of cylindrical gear is called gear pair with meshing ahead pitch point. Its real meshing line (from Figure 6-31 we can see involute cylindrical gear pair) is on the right side of the Pitch (P), and this is why it is called gear pair with meshing ahead pitch point.

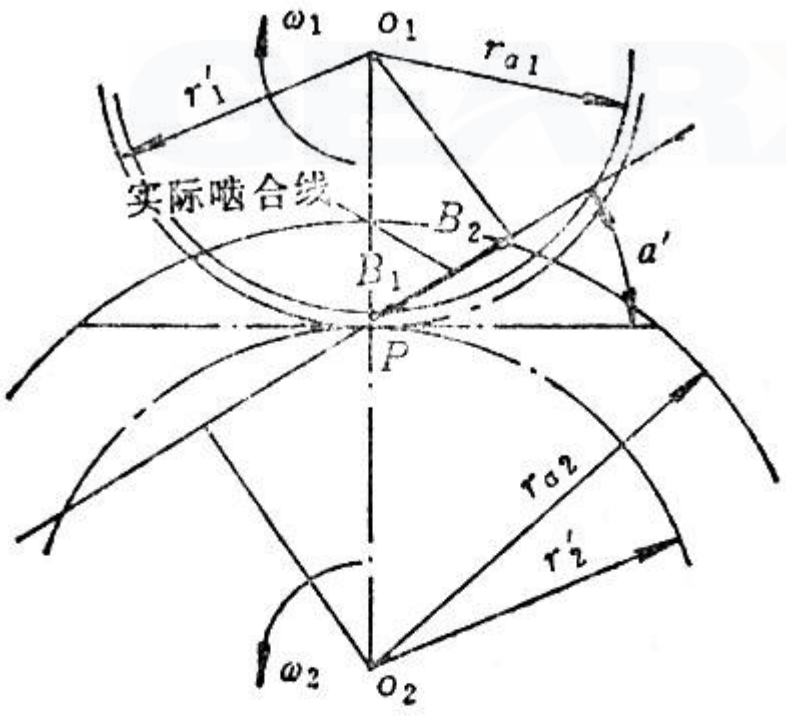


图 6-31

Gear pair with meshing behind pitch point

In cylindrical gear pair, if the significant tooth surface of driving gear is located between the pitch cylinder and addendum cylinder, while the significant tooth surface of driven gear is located between pitch cylinder and dedendum cylinder, then the real meshing line is located in the exit side of the pitch (P) during the meshing process. This kind of cylindrical gear pair is called gear pair with meshing behind pitch point. We can see involute cylindrical gear pair with meshing behind pitch point from the Figure 6-32.

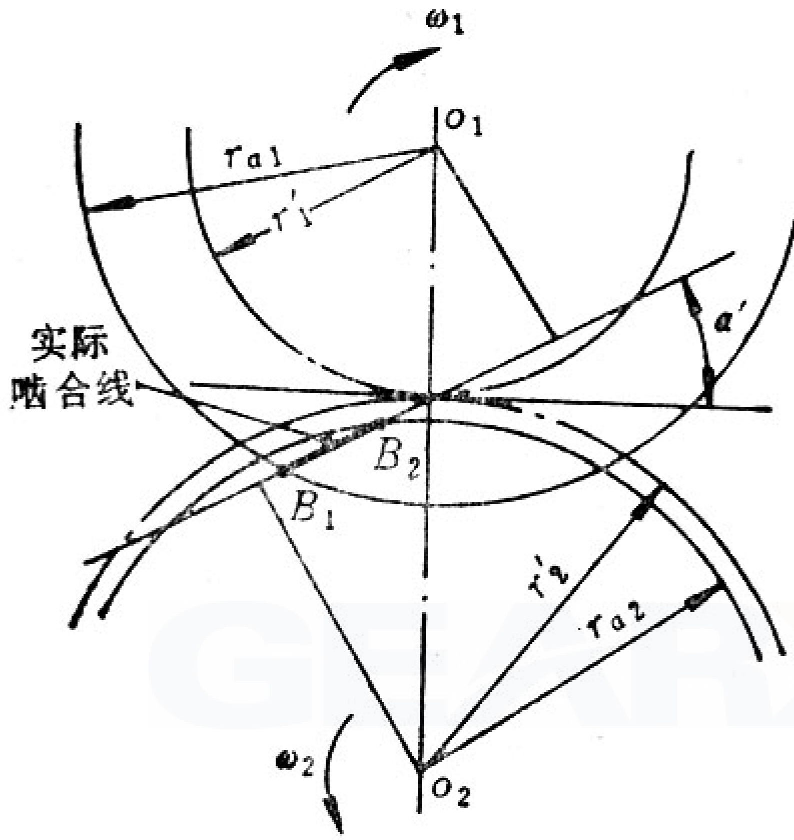


图 6-32

Modified gear pair of pitch point in double contact zone

In spur gear pair, during the whole meshing process of gear teeth in and out, with two phases of two pairs and single pairs both experienced, if modified gear pair is applied and there is concurring meshing of two pairs of teeth at the moment when pitch point is beginning to engage, then this kind of modified gear pair is called modified gear pair of pitch point in double contact zone (Figure 6-33). Nearby the pitch point, there is the weaken zone of teeth surface. Putting the pitch point in double contact zone can effectively improve the strength of tooth surface.

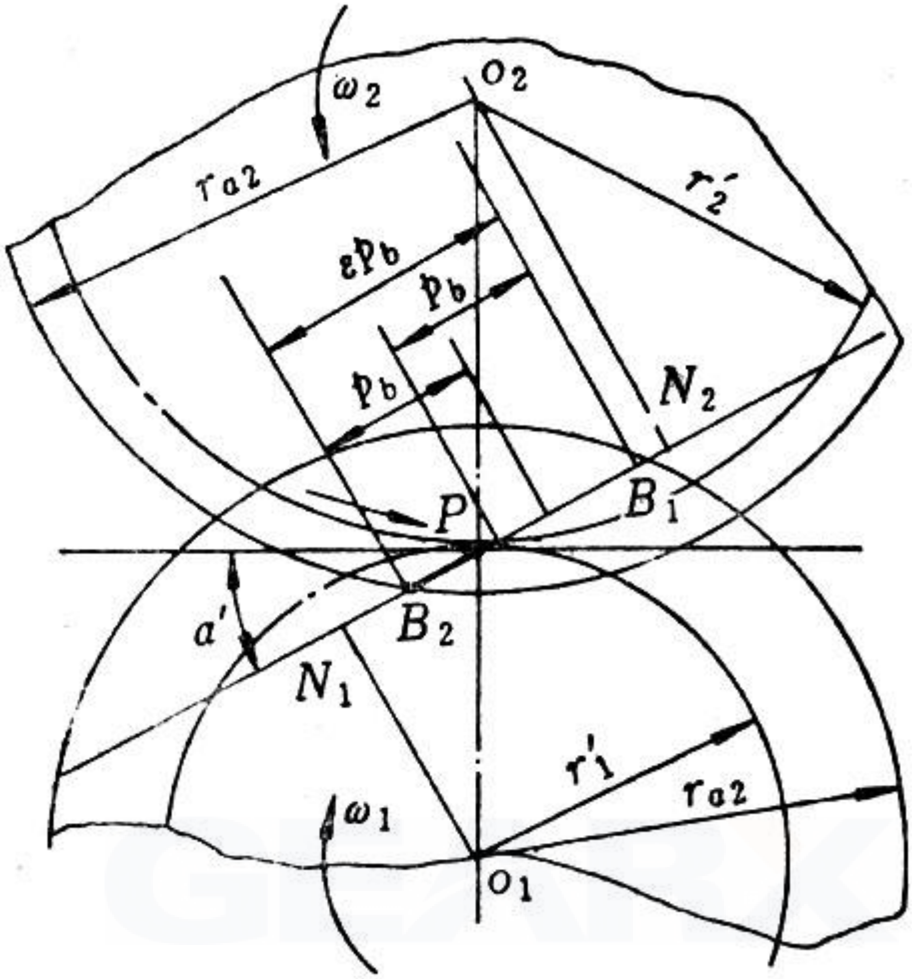


图 6-33

Geometric size drawing for involute cylindrical gear pair

This kind of drawing refers to involute cylindrical gear pair meshing drawing that clearing notes the geometric size or code name and the relations between them. Detailed can be seen from the Figure 6-34.

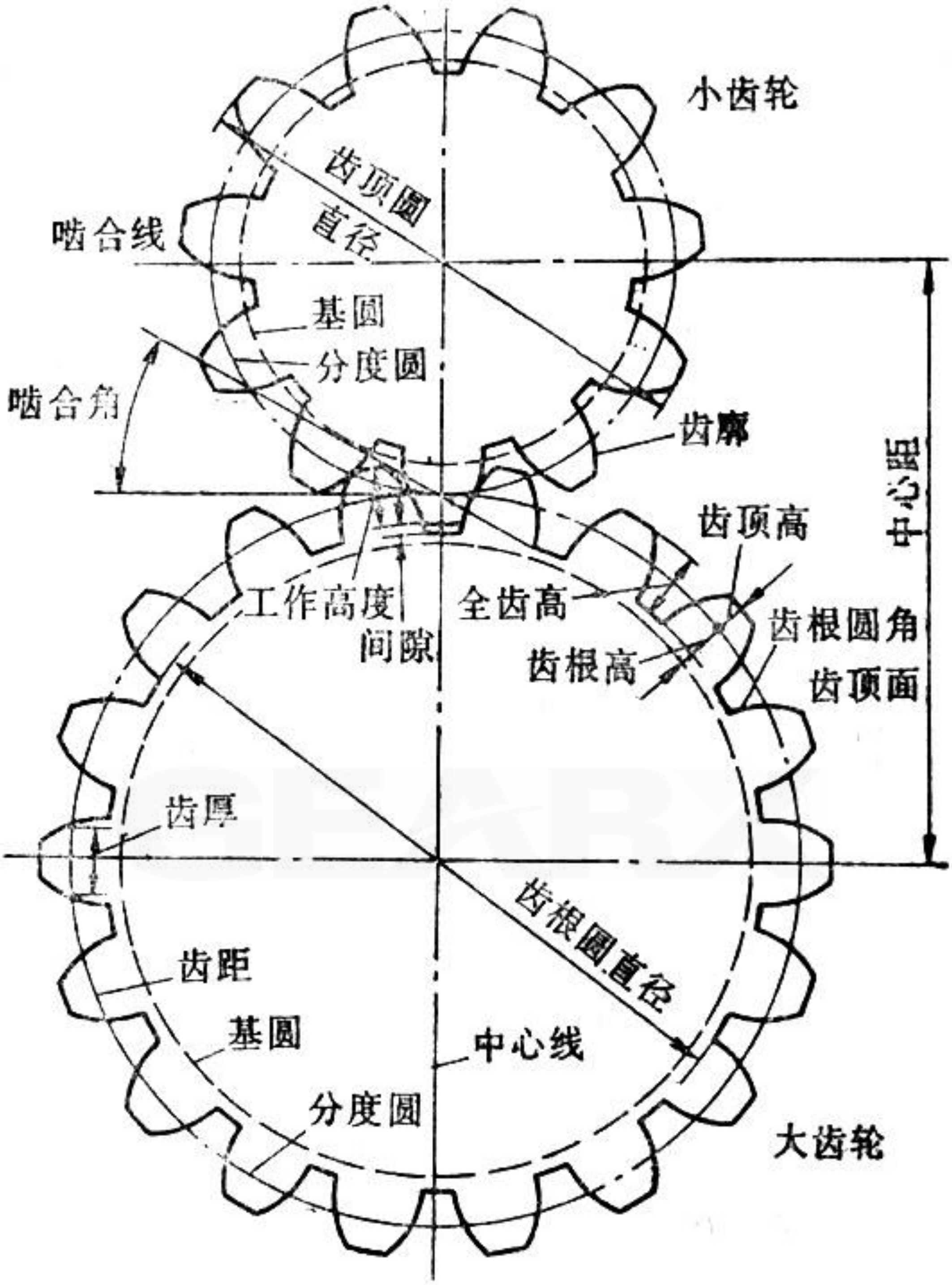


图 6-34

Speed reducing gear pair

The angular speed of driven gear is smaller than that of the driving speed, which refers to gear pair that taking pinion as driving gear (Figure 6-35). This is called speed reducing gear pair. The characteristic of this kind of gear is that its average transmission ratio is equal to the tooth number and it is larger than 1.

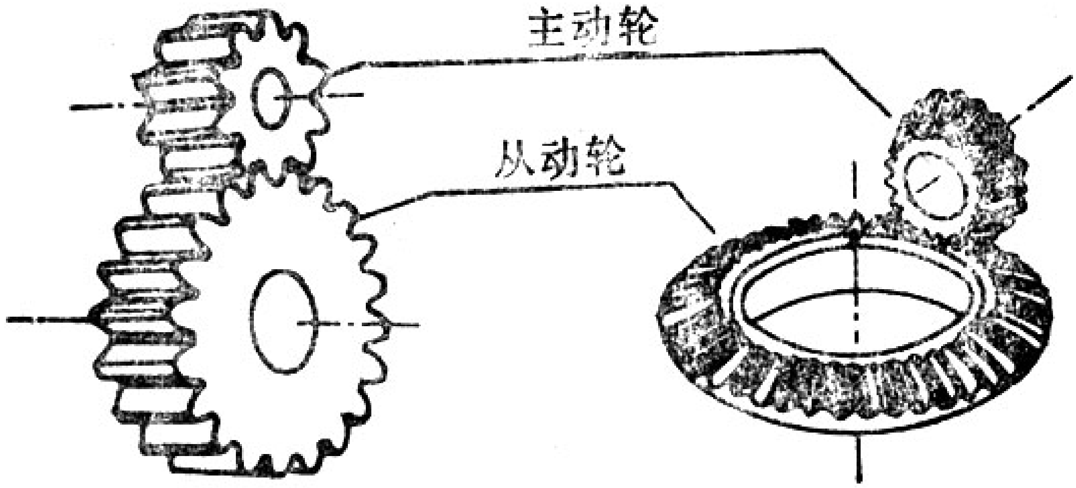


图 6-35

Speed increasing gear pair

The angular speed of driven gear is larger than that of the driving gear, which refers to gear pair whose bull gear serves as the driving gear (Figure 6-36). The characteristic is that the transmission ratio i_{12}<1 and the tooth number ratio u>l.

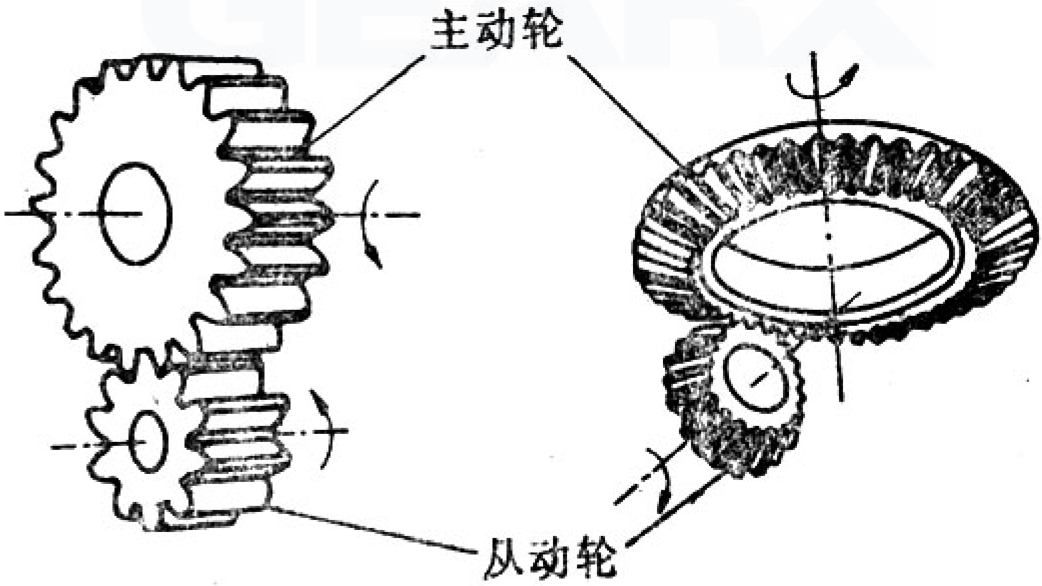


图 6-36

Conjugate gear pair

If the tooth profile of a pair of gear is conjugate tooth profile and they can operate stable and constantly in the whole process of engagement, then those pair of gears is called conjugate gear pair and the two gears are called conjugate gears.

Crowning gear pair

Crowning gear pair is consisted of a pair of conjugate crowning gears.

Sliding factor curve of involute gear pair

The limiting value of ratio formed between relative operation speed and tangent speed of conjugate tooth profile is called sliding factor. Curve drawn by sliding factor is called sliding factor curve.

The sliding factor of external involute gear pair is:

$$\begin{cases} \eta_1 = \dfrac{\overline{PK}}{\rho\kappa_1}\left(\dfrac{u+1}{u}\right) \\ \eta_2 = \dfrac{\overline{PK}}{\rho\kappa_2}(u+1) \end{cases}$$

The slip factor of internal gear pair is:

$$\begin{cases} \eta_1 = \dfrac{\overline{PK}}{\rho\kappa_1}\left(\dfrac{u-1}{u}\right) \\ \eta_2 = \dfrac{\overline{PK}}{\rho\kappa_2}(u-1) \end{cases}$$

When meshing at the point of N_l, K and N_l will overlap, $\rho\kappa_1=0$， $\eta_1=\infty$; when meshing at the point of N_2, $\rho\kappa_2=0$， $\eta_2=\infty$; When meshing at P point $\overline{PK}=0$, it means that the meshing point K and P will overlap, $\eta_1=\eta_2=0$. Then sliding factor curve can be drawn (Figure 6-37a, b).

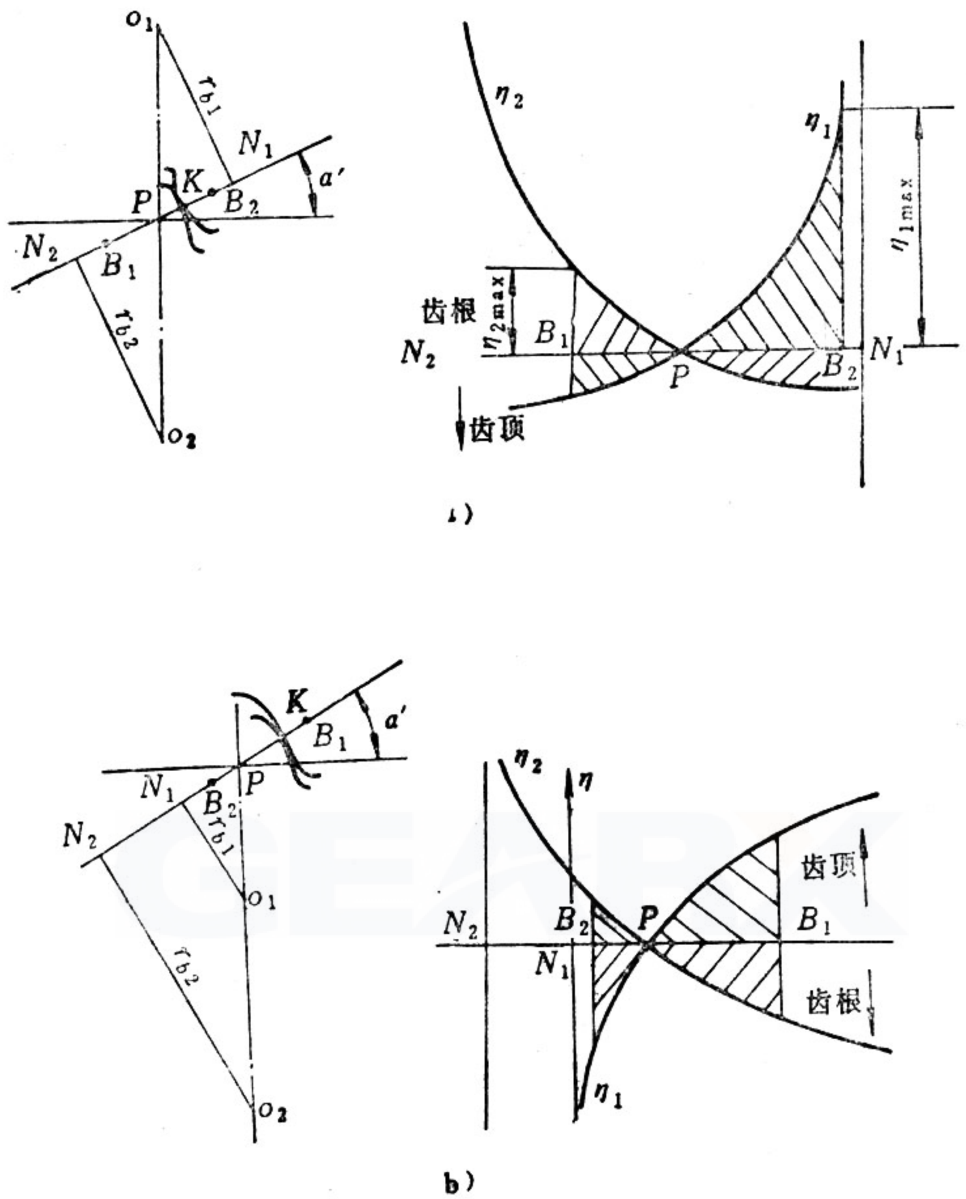


图 6-37

a）外齿轮副滑动系数曲线 b）内齿轮副滑动系数曲线

a) Sliding factor curve of external gear pair

b) Sliding factor curve of internal gear pair

From the figure we can know that slip factor is the function of meshing point at K; the slip factor of pinion and bull gear is not the same; $\eta_{1max} > \eta_{2max}$ pinion is easily get abrased. Even for the same gear teeth, the slip ratio of addendum and deddendum is not the same. In order to make less abrasion between tooth surfaces, η_1 and η_2 can approach to each other by modification.

Factor for geometric pressure of involute gear pair

For involute gear pair, the contact stress formed between conjugate tooth surfaces is a critical factor used to measure the surface strength of gear tooth. Contact stress $\sigma_{H\max} = \sigma_H = 0.418\sqrt{\frac{F_t E}{L_{\min} \cdot \alpha'} \cdot \left(\alpha' \sqrt{\frac{tg\alpha'}{\rho_1 \cdot \rho_2}}\right)}$ can be calculated through Hertz formula. In the formula, $\left(\alpha' \sqrt{\frac{tg\alpha'}{\rho_1 \cdot \rho_2}}\right)$ is the geometric pressure factor ψ. This coefficient indicates the influence of main engagement parameter a', α', the position of meshing point and geometric parameter ρ_1, ρ_2 on contact stress. When involute gear pair engage at the engagement pole N_1 (or N_2), ρ_1 (orρ_2) is zero, and the geometric pressure factor is an infinitely big ∞, and they are meshing on the central point of theoretical meshing line. If we make the overlap ratio ε=1, then the geometrical pressure factor will be the minimum. Geometrical pressure factor curve can be classified into theoretical geometric factor curve and actual geometric factor curve. Here it refers to the former. Details can be seen from the Figure 6-38a, b.

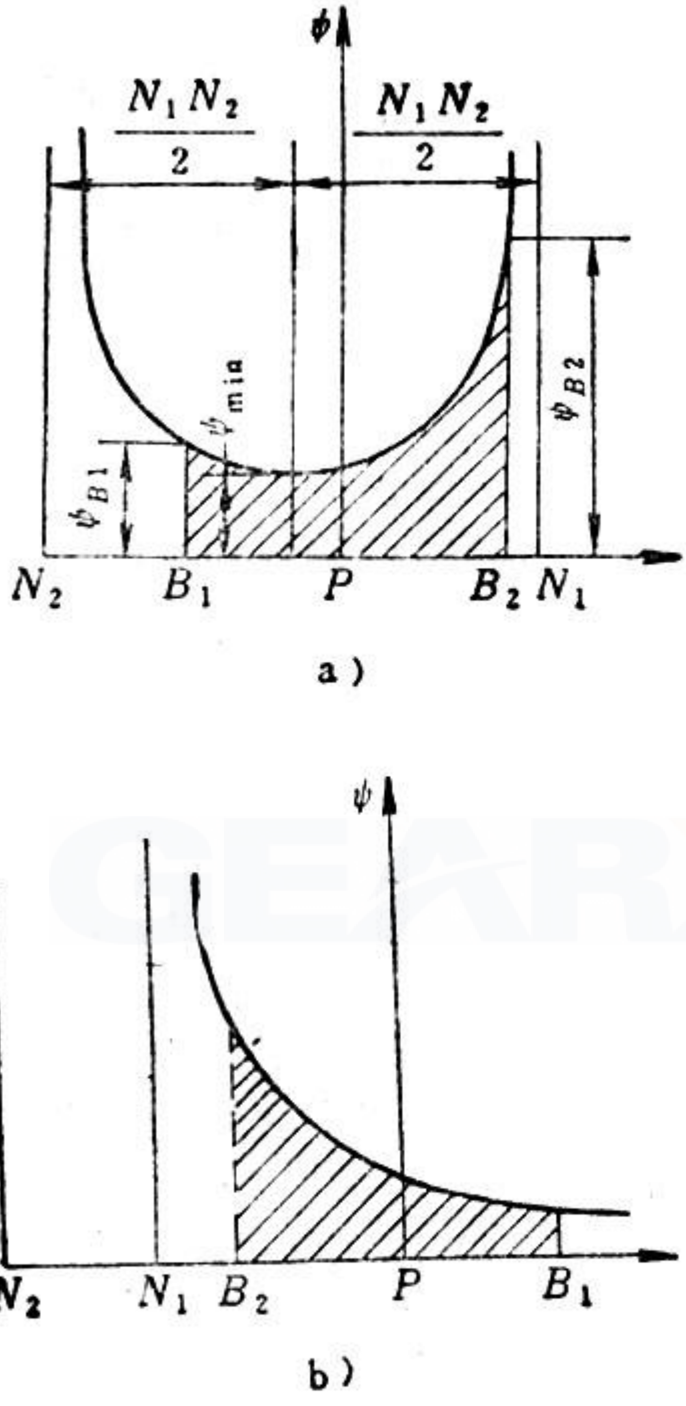


图 6-38

a）外啮合齿轮传动的几何压力系数

b）内啮合齿轮传动的几何压力系数

a) Geometric pressure coefficient of external gearing transmission

d) Geometric pressure coefficient of internal gearing transmission

Actual geometric pressure factor of involute gear pair

In actual working process, the overlap ratio of gear pair $\varepsilon > 1$, and it works in the sector of actual meshing line. At the time, considering the influence of contact ratio on tooth surface pressure, the geometric pressure factor drawn on the actual meshing line $\overline{B_1B_2}$ is the actual geometric pressure factor curve. This curve roughly reflects the real situation of tooth surface pressure. When $\varepsilon=1$, the geometric pressure factor curve drawn has minimum pressure at P point. When $\varepsilon > 1$, generally, pressure beard at P point is approaching to the maximum.

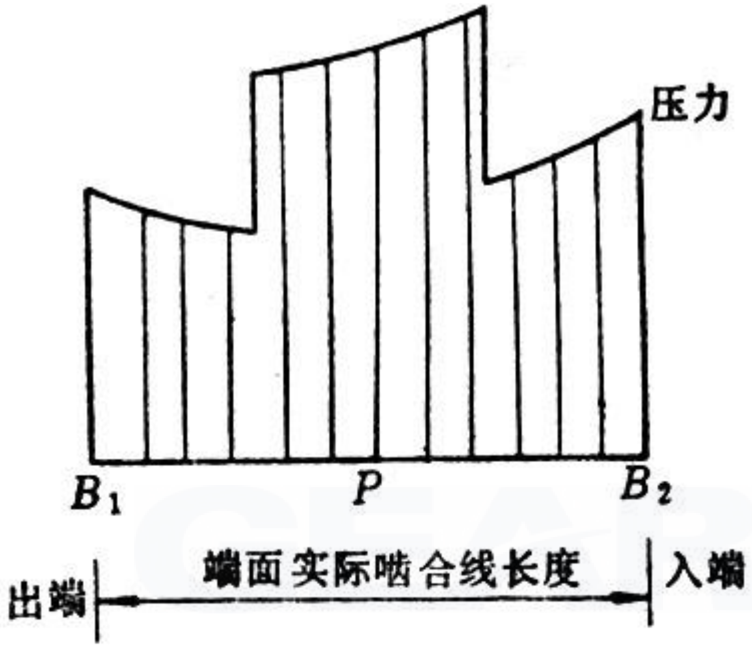


图 6-39

Factor for geometric pressure of involute gear in pitch point

For involute gear pair, the contact stress formed between conjugate tooth surfaces is a critical factor used to measure the strength of gear tooth surface. Factor for geometric pressure $\psi = \alpha' \sqrt{\frac{tg\alpha'}{\rho_1 \cdot \rho_2}}$ can reflect surface stress. It is the function for the position of meshing point. If we put (at the point of P) the radius of curvature of tooth profile ρ_{1P},ρ_{2P} into the formula, we can get the factor for geometric pressure $\psi_P = (z_2 \pm z_1)\sqrt{\frac{2}{z_1 z_2 \sin 2\alpha'}}$ of involute gear at the pitch point P. We can learn from the formula that the larger the meshing angle α' is, the smaller Ψ_P is and the higher the bearing capacity of tooth surface is. Therefore, the ratio $\frac{\sin 2\alpha'}{\sin 2\alpha}$ can be used to assess roughly the surface strength relation between modified gear pair and standard gear pair.

Curve of equal slip ratio of involute gear pair

Curve drawn under the condition of $\eta_{1max}=\eta_{2max}$ (the slip ratio of two engaged gear teeth) with modification method is called curve of equal slip ratio (Figure 6-40). For external gearing involute gear pair:

$$\begin{cases} \eta_{1\max} = \dfrac{tg\alpha_{a_1} - tg\alpha'}{\left(1+\dfrac{z_1}{z2}\right)tg\alpha' - tg\alpha_{a2}}\left(\dfrac{u+1}{u}\right) \\ \quad = \eta_{2\max} = \dfrac{tg\alpha_{a1} - tg\alpha'}{\left(1+\dfrac{z_2}{z_1}\right)tg\alpha' - tg\alpha_{a1}}(u+1) \\ x_{\Sigma} = x_1 + x_2 = \dfrac{z_1+z_2}{2\mathrm{tg}\alpha}(\mathrm{inv}\alpha' - \mathrm{inv}\alpha) \end{cases}$$

x_1、x_2 calculated from the above formula are radial modification factors that can ensure $\eta_{1max}=\eta_{2max}$. We can, therefore, draw the curve of equal slip ratio. It is the function of tooth number ratio u. Different u corresponds to different curve. This curve is a quality index curve in the closed graph.

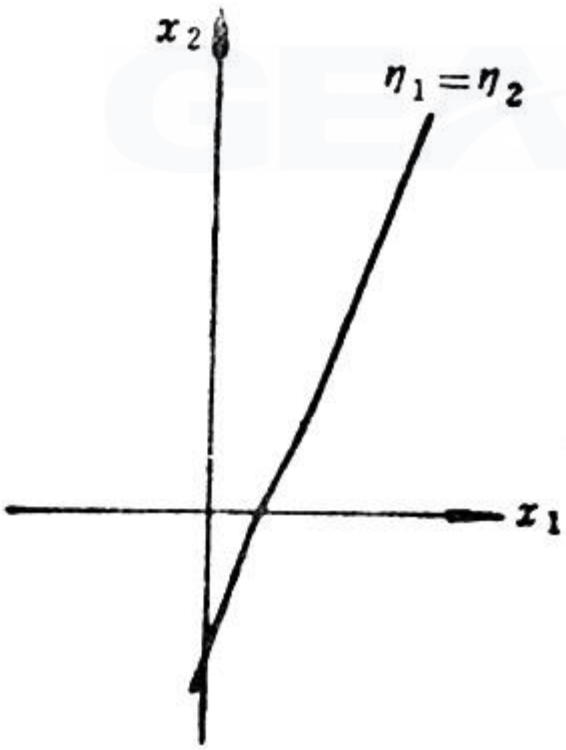


图 6-40

Involute interference

For gear pair of external gearing, if the tooth top of pinion is going beyond the engagement pole of the bull gear, or the tooth top of bull gear goes beyond the engagement pole of the pinion, then gear pair of external gearing will have involute interference during the process of engagement. If taking the pinion as gear planer cutter, generating undercutting or generating top cut would occur.

With regard to internal gear pair, if the addendum circle of internal gear goes beyond the engagement pole of external gear (Figure 6-41), interference would also occur.

The interferences above are all called involute interference.

The condition of non-interference for external gear pair is:

$$z_{2\max}=\sqrt{\frac{(z_1+2h_a^*+2x)^2-(z_1\cos\alpha)^2}{\sin\alpha}}-z_1$$

The condition of non-interference for standard internal gear pair ($x_1=x_2=0$) is:

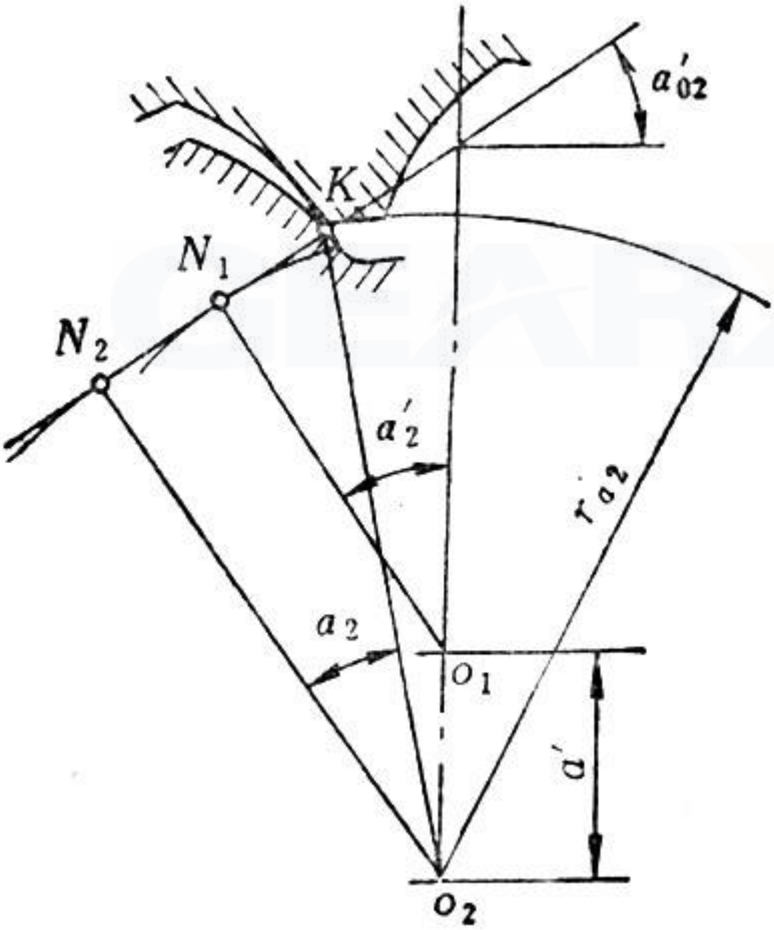
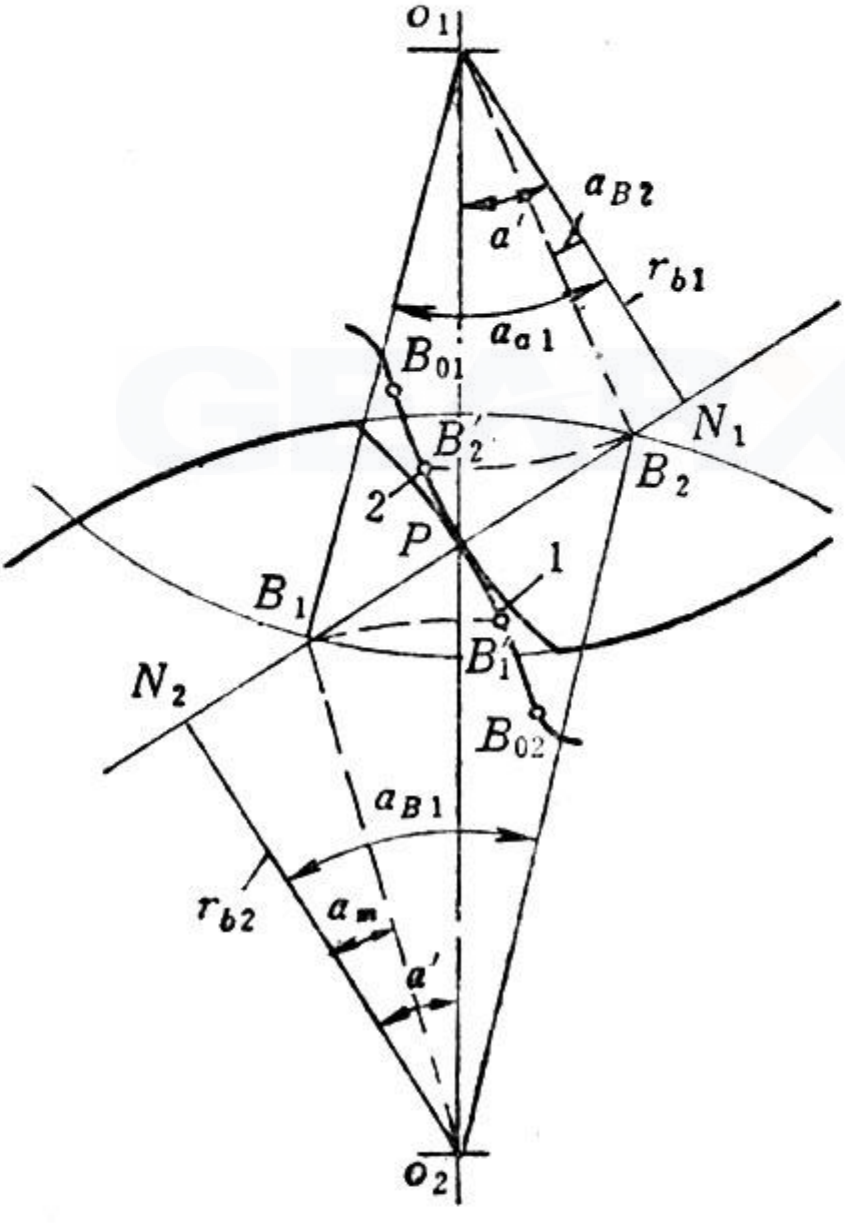


图 6-41

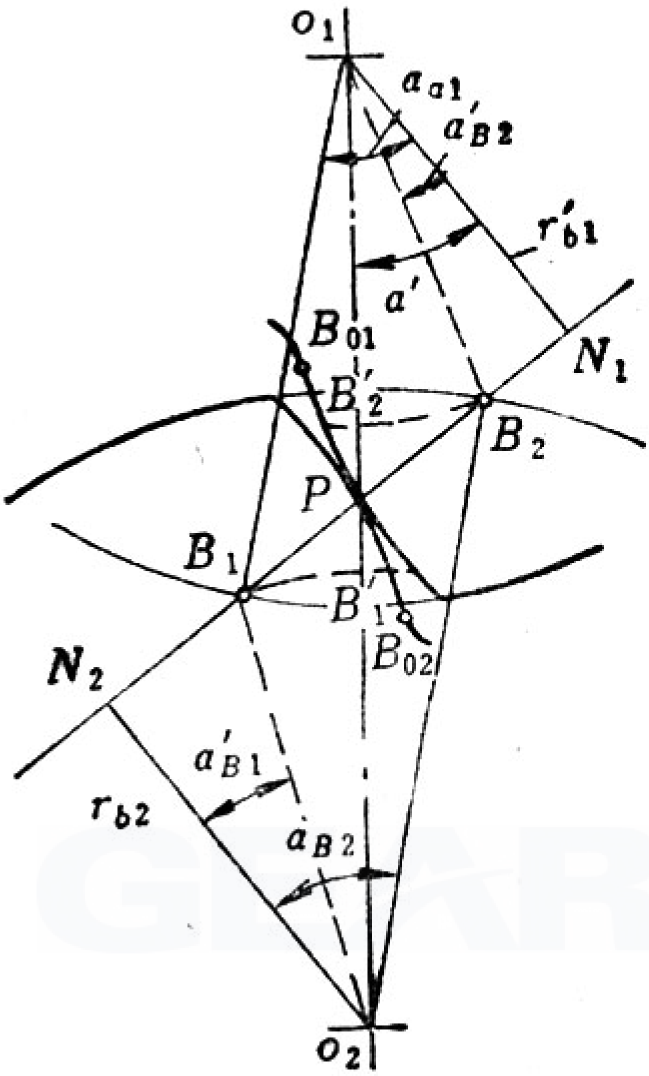
$$z_2 \geq \frac{z_1^2 \sin^2\alpha - 4(\frac{h_{a2}}{m})^2}{2z_1 \sin^2\alpha - (\frac{2h_{a2}}{m})^2}$$

Interference of transition curve (or fillet) of external meshing gear pair

For a pair of involute gears of external gearing, if the involute tooth profile of one gear at tooth top and the tooth profile of non-involute transition curve of another gear at the root of the tooth are meshing together (Figure 6-42), the conjugate meshing condition will lose and cause the change of transmission ratio and may even result in stuck phenomenon. This is what we called interference of transition curve of external gearing gear pair.



a)



b）

图 6-42

a）一对用齿条刀具加工的齿轮的啮合

b）一对用插齿刀加工的齿轮的啮合

a) The engagement of a pair of gears manufactured with rack and cutting tool

b) The engagement of a pair of gears manufactured with gear planer cutter

For a pair of external gears cut by rack cutter, the condition for not happening interference of transition curve is: the point on the transverse tooth profile of a gear, conjugating with the tip point of another gear, mush fall on the involute profile line. This means that the pressure angle at this point must larger than that at the starting point of involute tooth profile. Then, we can learn that the condition for not occurring interference of transition curve is:

For gear 1:

$$tg\alpha' + \frac{z_2}{z_1}(tg\alpha' - tg\alpha_{a2}) \geq tg\alpha - \frac{4(h_a^* - x_1)}{z_1 \sin 2\alpha};$$

For gear 2:

$$tg\alpha' + \frac{z_2}{z_1}(tg\alpha' - tg\alpha_{a1}) \geq tg\alpha - \frac{4(h_a^* - x_2)}{z_2 \sin 2\alpha}。$$

The condition of not having interference of transition curve for a pair of gears of external gearing cut by gear shaper cutter is: the pressure angle at the starting point of tooth profile must be larger than that at the starting point of involute line. Then, we can learn that the condition for not occurring interference of transition curve is:

For gear 1:

$$tg\alpha' + \frac{z_2}{z_1}(tg\alpha' - tg\alpha_{a2}) \geq tg\alpha_0' + \frac{z_{01}}{z_1}(tg\alpha'_{01} - tg\alpha_{a0});$$

For gear 2:

$$tg\alpha' + \frac{z_1}{z_2}(tg\alpha' - tg\alpha_{a1}) \geq tg\alpha_{02}' + \frac{z_{02}}{z_2}(tg\alpha_{02}' - tg\alpha_{a0})$$

Most number of teeth of non-topping for standard gear

This means there is no topping phenomenon on processing gear with rack cutter. When cutting gear with slotting tool, if there are too much teeth on the manufactured standard gear, its addendum circle would goes beyond the engagement pole N_0 on the base circle of gear slotting tool. At that time, parts of the involute tooth profile at the top of the manufactured gear are cut. This is called topping (Figure 6-43).

The condition of non-topping is: the tip circle of the manufactured standard gear not goes beyond the engagement pole N_0 of the base circle of shaper cutter, then:

$$z_{max} = \frac{z_0^2 \cos^2 \alpha \, tg^2 \alpha_0' - 4h_a^*}{4h_a^* - 2z_0 \cos^2 \alpha \, tg^2 \alpha_0'}$$

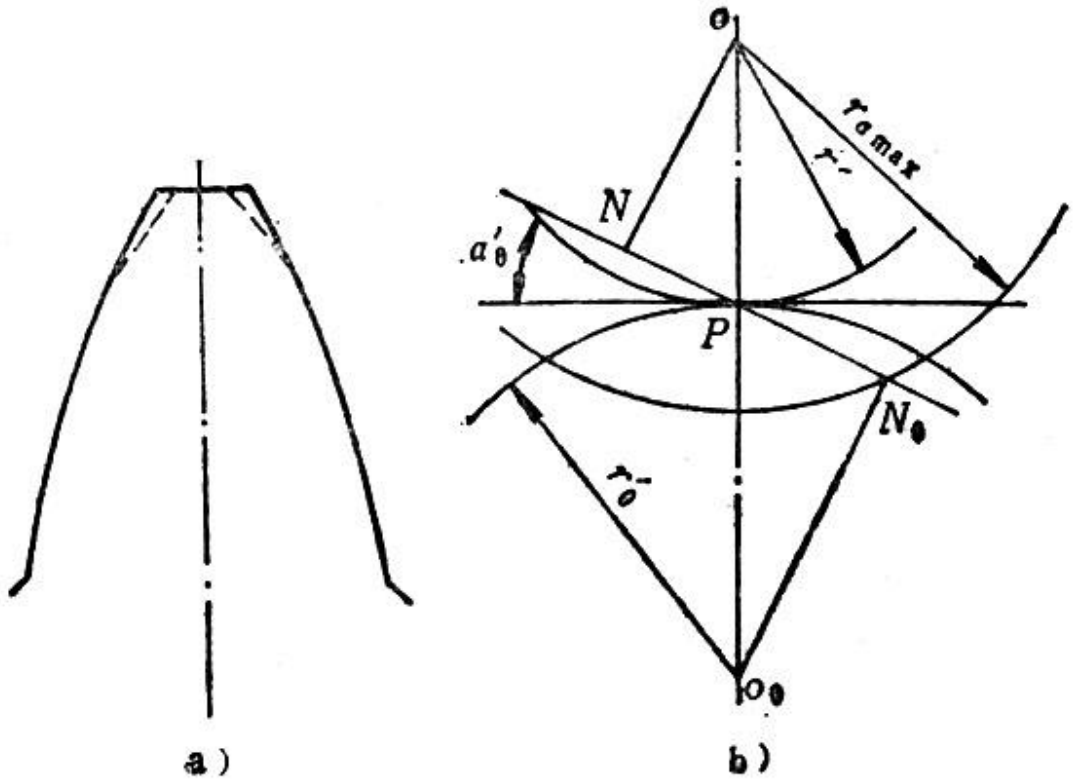


图 6-43

6.2.2 Internal gear pair

Internal gear pair

Internal gear pair is composed of one external gear and one internal gear. It is conjugate gear pair (Figure 6-44). Generally, it is used in planetary gear train. The common internal gear pair are: internal gear pair with parallel axes (including spur tooth, helical tooth and herringbone tooth); internal gear pair with intersecting axes; internal gear pair with alternating shaft (rarely used). Compared with external gear pair, internal gear pair has many following features: compact structure, large radius of curvature, large overlap ratio, more instantaneous meshing teeth, smooth working, small noise; small slip factor; large bearing capacity; currently, there are some difficulties in processing internal gear because of its strict design requirements. If not following the strict rules, interference phenomenon may occur frequently.

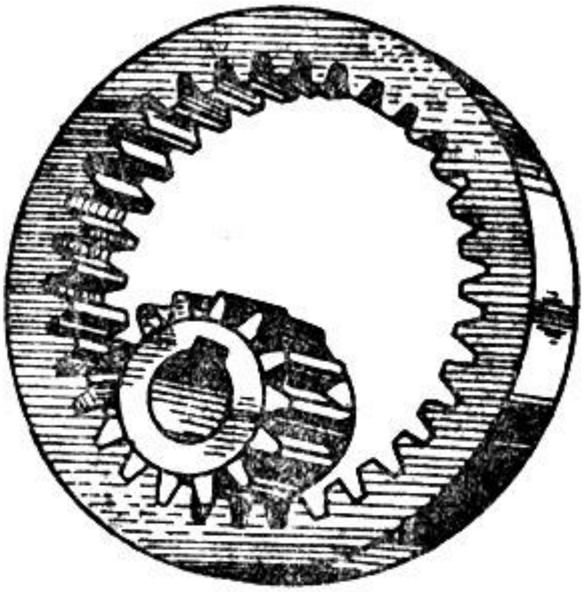


图 6-44

Involute spur gear pair of internal gearing with parallel axes

It is gear pair with parallel axes composed of an involute external gear and a matched involute internal gear. Except all the engagement features of involute gear pair with parallel axes, the characteristics of involute spur gear pair of internal gearing with parallel axes are: concave-convex engagement of conjugate tooth surface, large radius of curvature and strong contact strength; small slip factor, good lubrication condition, strong anti-scuffing capacity and high wear resistance; it has many engagement teeth, large overlap ratio small noise and working in a smooth way. But the design is relatively complicated. There may have meshing interference, tooth cutting interference or installation interference when parameters are chosen wrong.

Involute helical gear pair of internal gearing with parallel axes

It is cylindrical gear pair of internal gearing with parallel axes consist of matched external involute helical gear and involute internal helical gear. Details can be seen from “involute gear pair with parallel axes” and “helical internal gear pair”.

Herringbone gear pair of internal gearing

Herringbone gear pair of internal gearing is composed of a matched herringbone external gear and a herringbone internal gear. Generally, it refers to “involute herringbone gear pair of internal gearing with parallel axes”. It has all the meshing characteristics of helical internal gear pairs and can autonomously balance the axial force during its working process. The engagement characteristics of helical gears are very notable because larger helix angle is adopted in herringbone gear.

Involute internal gear pair with few tooth difference

This kind of internal gear pair consists of a pair of engaged involute gears with few tooth difference (z_2-z_1=l～4). It is usually used in involute planetary gear with few tooth difference and rarely used independently in internal gear pair with few tooth difference, because it cannot get large transmission ratio and not easy in design, manufacture as well as installation.

When involute internal gear pair with few tooth difference applied in involute planetary gear driving with few tooth difference, the smaller tooth number difference $\Delta z=z_2-z_1$ is, the larger the transmission ratio will be. For involute internal gear pair $h_a^*=1$、a=20°, when Δz keep smaller to some value, interference phenomenon would occur. In order to avoid interference and make the gear pair works normally, three methods are often adopted when designing the internal gear pair with few tooth difference: first, use short tooth, which means to make $h_a^*<1$. The common tooth form reference is a=20°, c^*=0.3～0.45; Second, properly adopt modification method and modification factor; third, increase the tooth profile angle of tool. It is rarely used because it is hard to change the tooth profile angle since the tool is a standard reference. Among these three, the second method is the most popular because it is more effective and rational. Detailed choice and design of specific modification method can be found through relative data. Various interference phenomenon of internal gear pair and ways to avoid them can refer to related items.

Internal gear pair with zero-meshing angle

In internal gear pair with zero-meshing angle, $r_{b1}=r_{b2}$, the overlap pitch point of two base circle is falling on the base circle, $r_{b1}=r_1'$,r_{b2}=r※. At this time, the angle formed between the meshing line and the common tangent is zero, which means the meshing angle $a'=0$. Besides, if the designed internal gear pair $r_{b1}=r_1'$, $r_{b2}=r_2'$, and the two base circles are tangent inside at P point, so the meshing angle at P is zero. From Figure 6-45, we can learn that tooth profile interference of internal gear pair is doomed to happen when the meshing angle is zero. Processing with gear shaper cutter would also bring top cut. In order to avoid tooth profile overlap interference when $a'=0$, external gear with negative modification and internal gear with positive gear or tangent modification are all can be adopted.

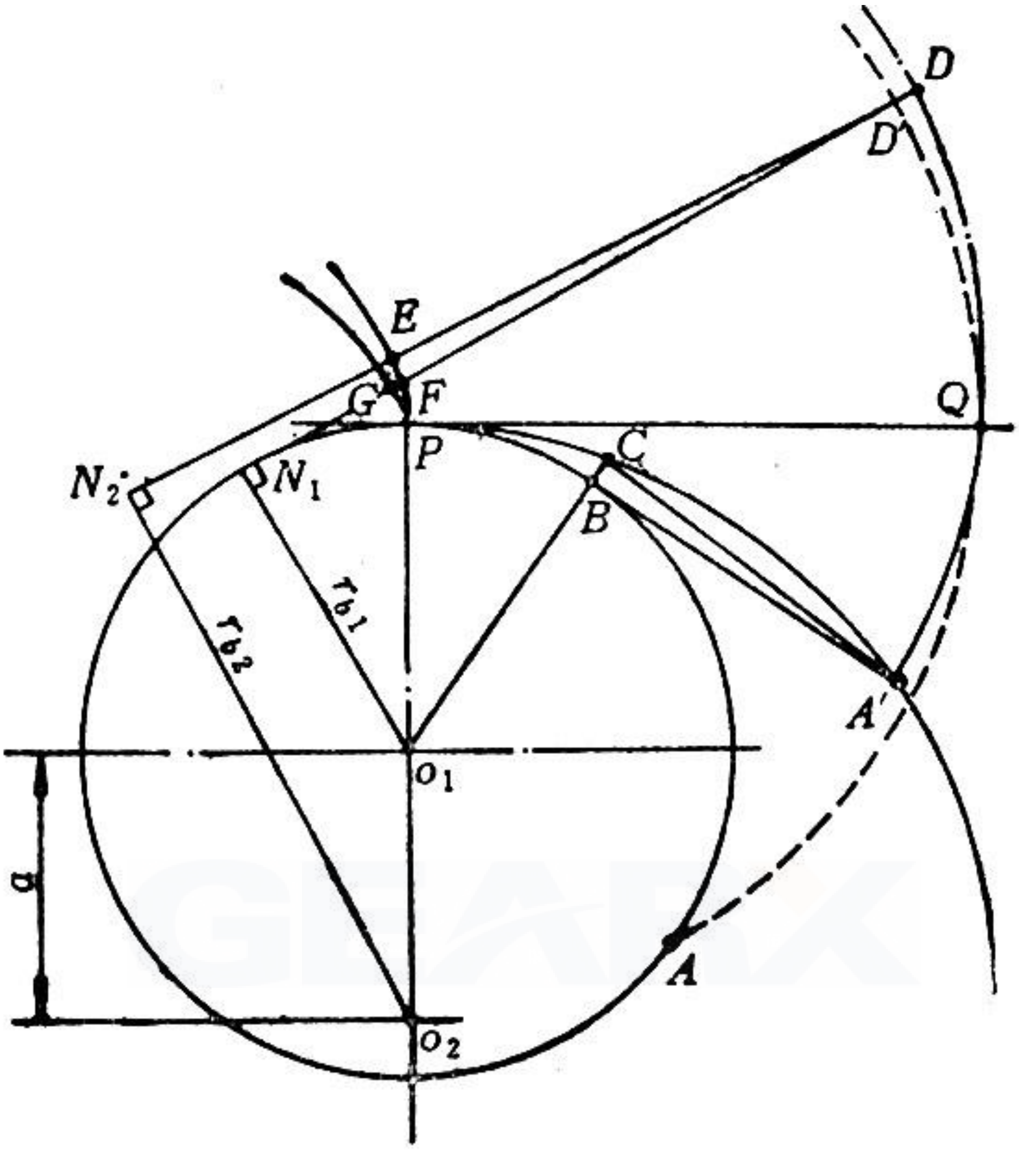


图 6-45

End face active conjugate profiles of internal gearing involute cylindrical gear pair with parallel axes

For involute cylindrical gear pair of internal gearing with parallel axes, the aggregate line on tooth surface formed by conjugate points in transverse plane is called effective conjugate tooth profile. Effective conjugate tooth profile can be calculated through drawing. From Figure 6-46, r_{a1} intersects with $\overline{N_1N_2}$ at B_1, taking $\overline{o_2B_1}$ as radius to draw a circle that intersects with the tooth profile of bull gear at a', then $\widehat{aa'}$ is called active tooth profile. At the same, tip circle r_{a2} intersects with $\overline{N_1N_2}$

at B_2, taking $\overline{o_1B_2}$ as radius to draw a circle and intersects it with the tooth profile of the pinion at b', then $\overset{\frown}{bb'}$ is called active tooth profile of the pinion.

Negative working pressure angle

When processing internal gear with gear shaper cutter, if there are too much tooth number on the gear shaper cutter, negative working pressure angle might occur. At that time, the two base circles of gear shaper cutter and internal gear is not intersecting with each other and no meshing line can be drawn. Under this circumstance, it is impossible to realize correct meshing, not mention processing right tooth form. Therefore, negative working pressure angle must be avoided. According to the equation of engagement with zero backlash, we can know the condition of no negative working pressure angle.

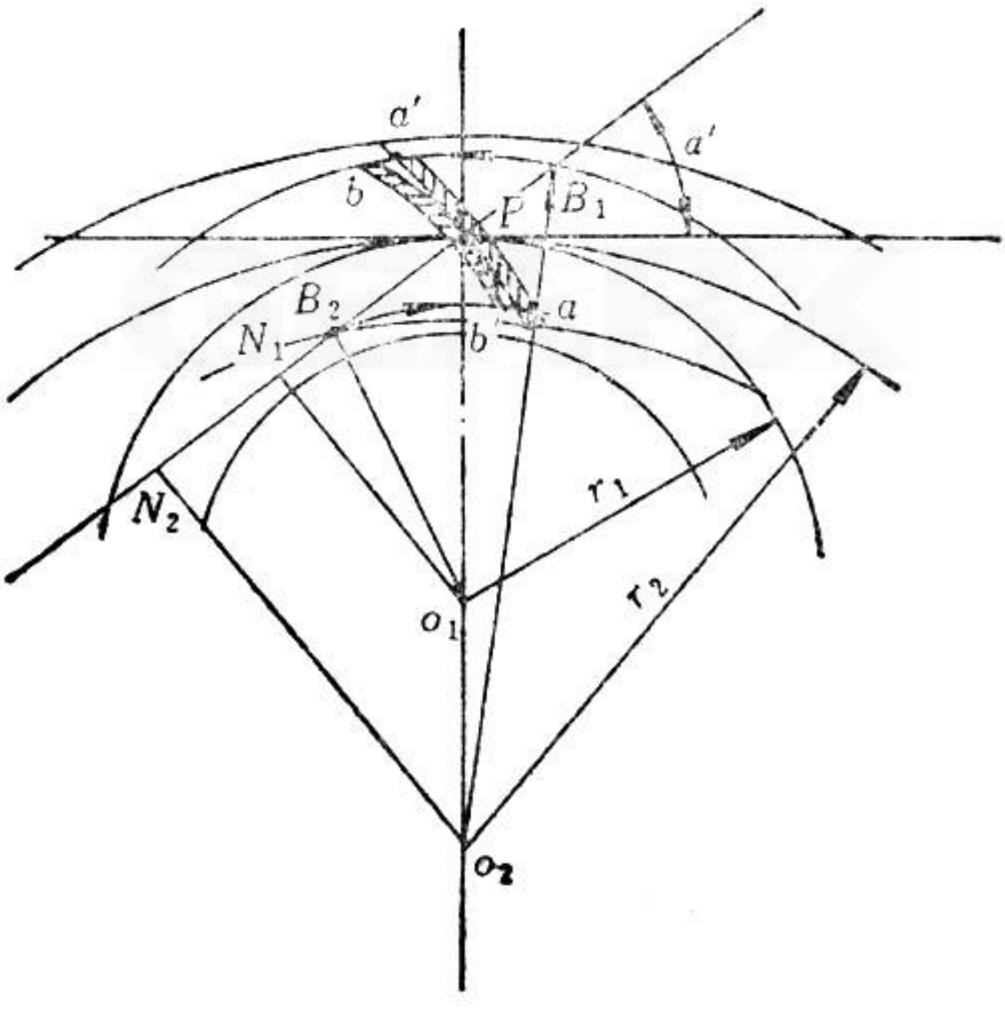


图 6-46

$$x_2 - x_0 \leq \frac{-(z_2 - z_1)\text{inv}\alpha}{2\text{tg}\alpha}$$

Radial modification of involute internal gear

At present, most of the involute internal gears are cut by gear planer cutter (Figure 6-47a). During the cutting process, radial modification of involute internal gear can be cut if the position of tool and gear blank being changed relatively. In fact, gear planer cutter is a modified gear. Different modification factor exists in different transverse surface (cutting face). Cutting face, in which x_0=0, is called original cross section. There is a rule: processing gear with gear planer cutter with original cross section, when we move the position of tool from processing standard gear to departing from the center of the manufactured gear. If the center distance of machine tool enlarge, then the modification factor is a positive, otherwise it is a negative. The internal gear manufactured is called positive modification gears and negative modification gear. Compared with processing external gear, when processing internal gear, the positive modified internal gear will decrease rather than increase because of the main difference between tooth thickness and common normal.

To calculate easily, people always taking the tooth space of internal gear as the tooth of external gear, and taking the modification factor of this imaginary external gear when manufactured with rack tool as the modification of internal gear (Figure 6-47c). Then relative formulas and graphs of external gear can be used for calculation. But the deddendum of the tooth, the diameter of deddendum circle is determined by the center distance a_0 of machine tool when processing with gear planer cutter.

Negative meshing of internal gear pair

With regard to the engagement between internal gear pair or gear shaper cutter and gear blank, if its engagement angle a'(or a_0') < 0, then it is called negative meshing of internal gear pair. The result of this phenomenon is that the base circle of the two gears (or tool and tooth blank) not intersects with each other and there is no common tangent. Meshing line would not form for the involute gear and it cannot work normally. Therefore, if negative meshing is not allowed, the following condition must be satisfied, which is a'(or a_0')>0, at this time:

$$\begin{cases} z_2 \geq z_1 + \dfrac{2\mathrm{tg}\alpha}{\mathrm{inv}\alpha}(x_1 - x_2) \\ x_2 - x_1 \geq -\dfrac{(z_2 - z_1)\mathrm{inv}\alpha}{2\mathrm{tg}\alpha} \end{cases}$$

$$\begin{cases} x_2 - x_0 \geq \dfrac{(z_2 - z_0)\mathrm{inv}\alpha}{2\mathrm{tg}\alpha} \\ x_2 - x_0 \geq 0.02047(z_2 - z_0) \end{cases}$$ (Note: it is used when α =20°)

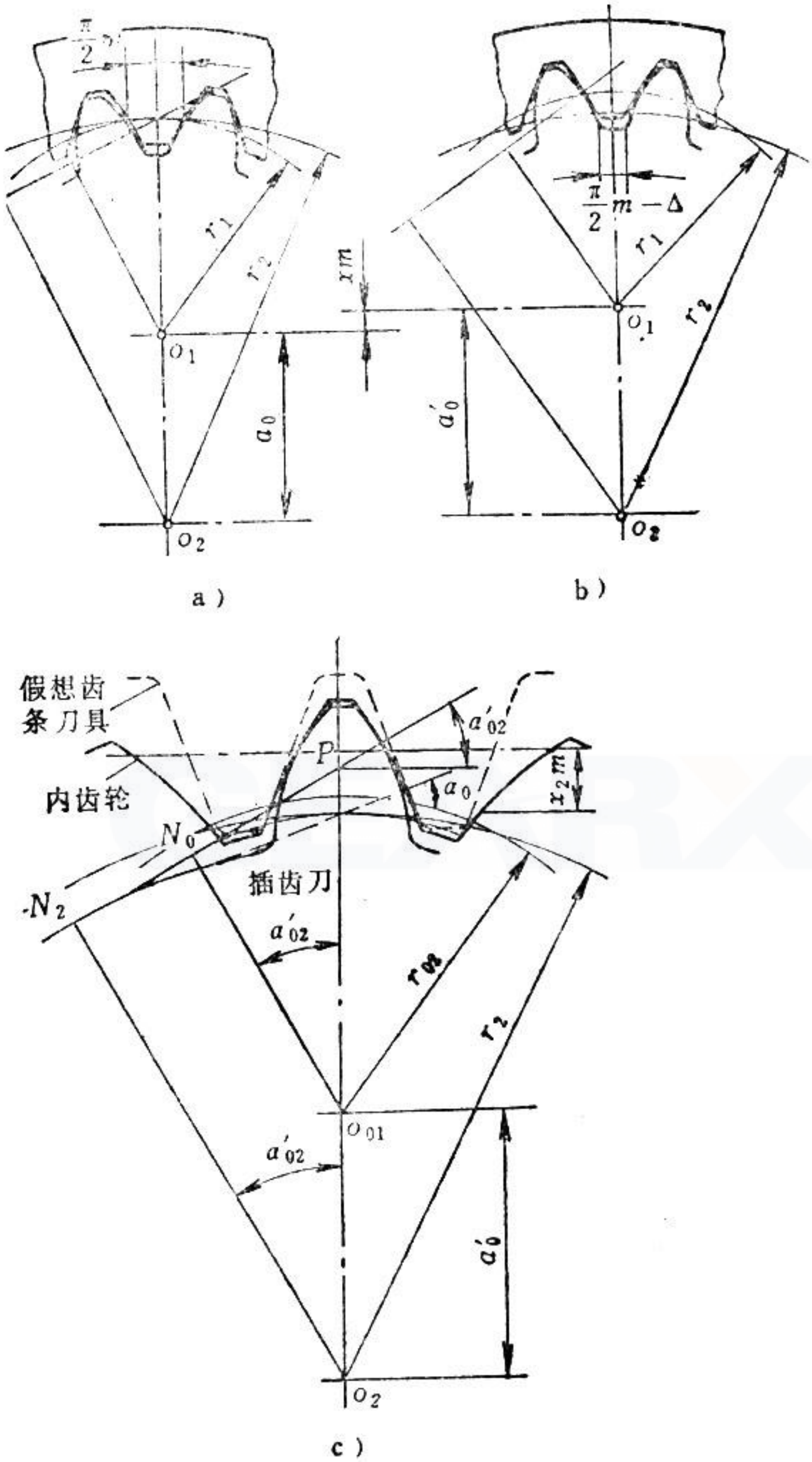


图 6-47

a）标准内齿轮副　b）变位内齿轮副

c）假想齿条刀具

a) Standard internal gear pair b) modified internal gear pair c) imaginary rack cutter

Geometric size drawing for involute inner gear pair

This kind of drawing indicate the sizes and names (they can be showed as code name) of involute inner gear. The drawing can also clearly reveal their relation.

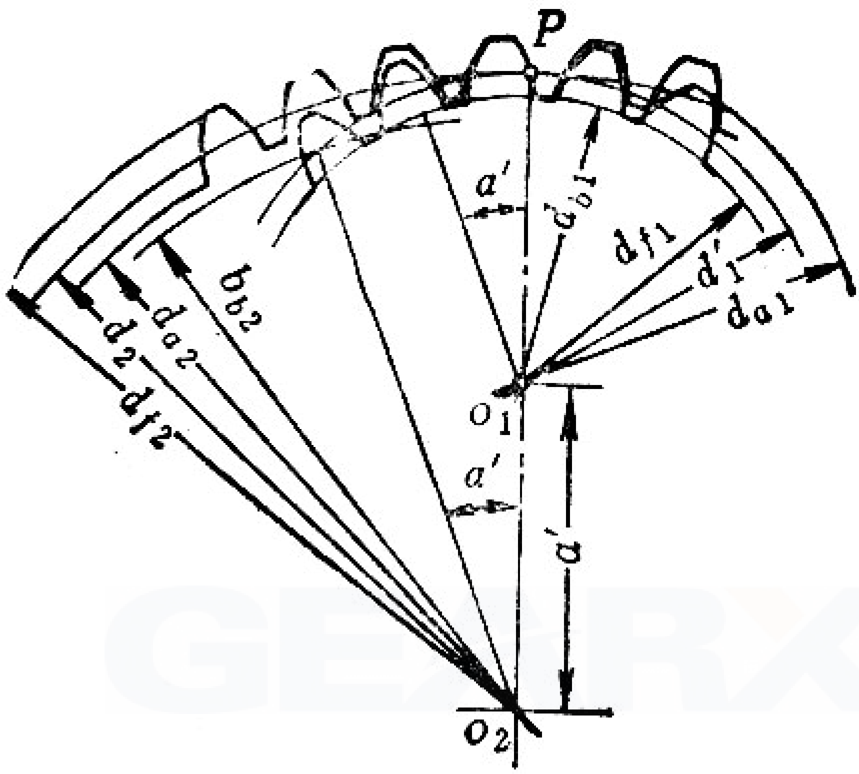


图 6-48

Interference of transition curve (or fillet) of gear pair with internal meshing

During the meshing process of internal gear pair, the tooth top of pinion may contact the transitional curve of inner gear, which breaks the condition of conjugate meshing because of ill design. This phenomenon is called interference of transition curve. The condition to avoid transitional curve interference is: the pressure angle at the end point of involute tooth profile of inner gear should be larger or equal to the pressure angle at the end point of working section tooth profile. Then:

$$\mathrm{tg}\alpha'_{0_2} + \frac{z_0}{z_2}(\mathrm{tg}\alpha_{a0} - \mathrm{tg}\alpha'_{0_2}) \geq \mathrm{tg}\alpha' + \frac{z_1}{z_2}(\mathrm{tg}\alpha_{a1} - \mathrm{tg}\alpha');$$

$$z_0 \mathrm{tg}\alpha_{a0} + (z_2 - z_1)\mathrm{tg}\alpha'_{a2} \geq z_1 \mathrm{tg}\alpha_{a1}$$

Or $$+ (z_2 - z_1)\mathrm{tg}\alpha'$$

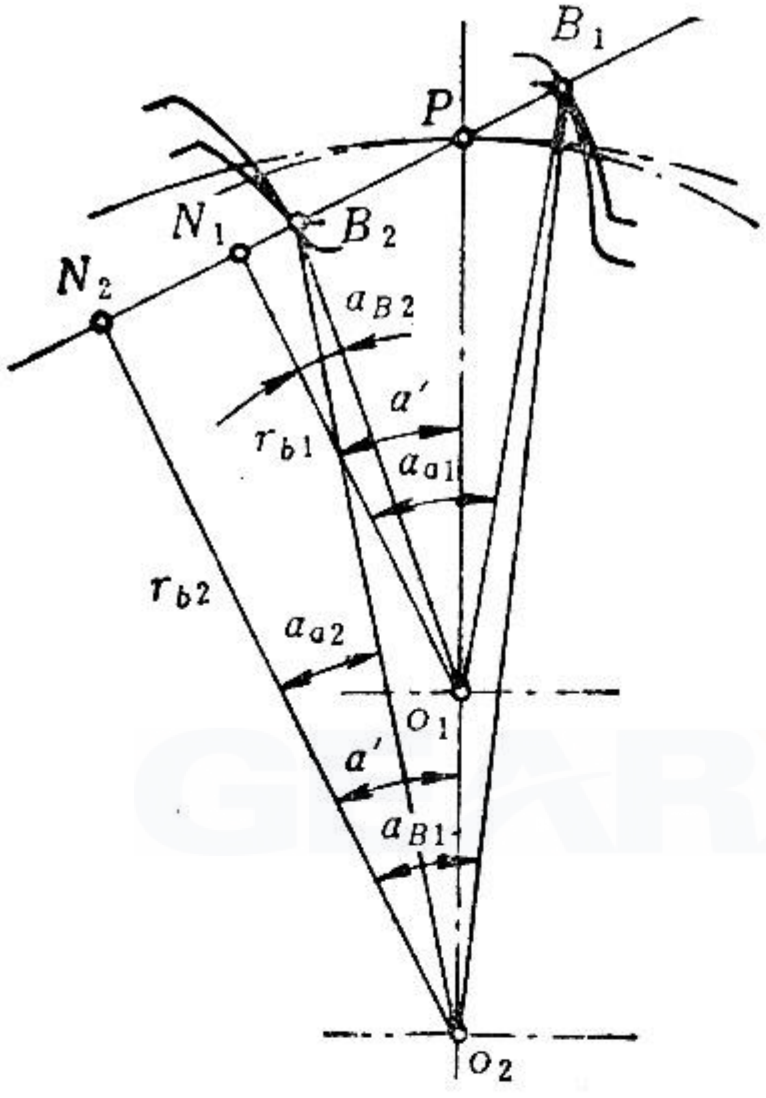


图 6-49

To avoid interference, increasing addendum modification coefficient of inner gear and decreasing the cutter addendum can be applied.

Pressure angle of end point of involute internal gear tooth profile

At present, most of the involute inner gear is manufactured by gear shaper cutter. From Figure 6-50, we can see that the gear shaper cutter cut the tooth profile of inner gear from point B_2 and end in B_1. And the whole part of involute tooth profile is processed. Here, transition curve refers to the area between the point B_1 to the deddendum surface. The point B_1 is called the end point of involute gear tooth profile of internal gear, and the pressure angle at this point is called pressure angle at the end point. The pressure angle of internal gear involute tooth profile at the end point is:

$$\mathrm{tg}\alpha_{B1} = \frac{1}{z_2}\left[z_0 \mathrm{tg}\alpha_{a0} + (z_2 - z_0)\mathrm{tg}\alpha_{a0}\right]$$

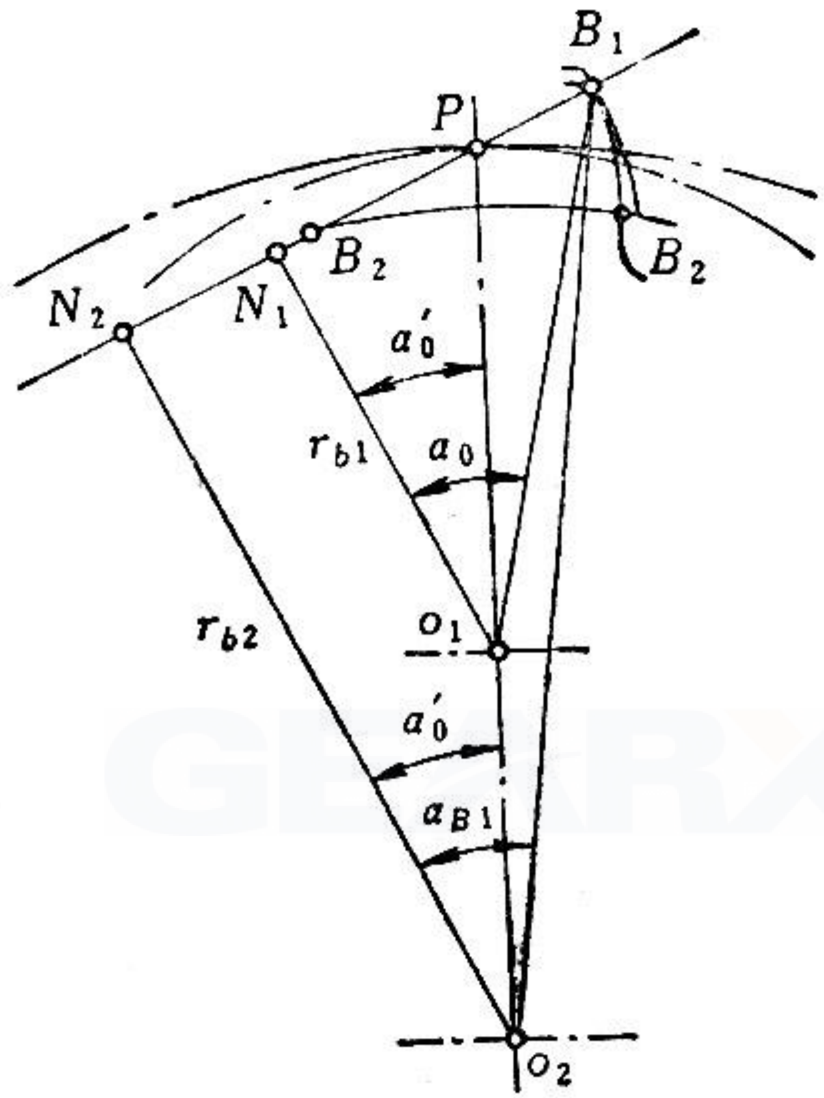


图 6-50

Trochoidal interference of internal gear pair

During the meshing process of internal gear pair, the top of pinion may contact the top of internal gear (Figure 6-51) when the former is exiting from the tooth space of internal gear because of the less difference of tooth number. This phenomenon is called trochoidal interference of internal gear pair. This may result in abnormal meshing as well as abnormal installation of internal gear pair. If pinion serves as gear planer cutter, then trochoidal interference may occur and there would be top cut, so trochoidal interference must be avoided.

The reason why it occurs is that the involute tooth profiles of another pair of gear teeth are intersecting when one pair of gears is under meshing process. Apparently, keeping tooth profile operates without intersection before the advent of intersecting top circle of two gears serves as one way to avoid trochoidal interference. From Figure 6-51b, we can learn the specific condition is $\angle ao_2P \geq \angle Mo_2P$. Therefore, the formula for not happening trochoidal interference is:

$$i_{21}(\delta_1 + \mathrm{inv}\alpha_{a1} - \mathrm{inv}\alpha') \geq \delta_2 + \mathrm{inv}\alpha_{a2} - \mathrm{inv}\alpha'$$

Or

$$z_1(\mathrm{inv}\alpha_{a1} + \delta_1) - z_2(\mathrm{inv}\alpha_{a2} + \delta_2) + (z_2 - z_1)\mathrm{inv}\alpha' \geq 0$$

In the formula,

$$\begin{cases} \delta_1 = \cos^{-1}\left(\dfrac{r_{a_2}^2 - \alpha'^2 - r_{a_1}^2}{2r_{a1}\alpha'}\right) \\ \delta_2 = \cos^{-1}\left(\dfrac{r_{a_2}^2 + \alpha'^2 - r_{a_1}^2}{2r_{a2}\alpha'}\right) \end{cases}$$

We can figure out that the critical factor that affects trochoidal interference is (z_2-z_1). The smaller the value is, the more likely trochoidal interference would happen. When (z_2-z_1)≥10, trochoidal interference phenomenon would not occur. When z_2-z_1 is small, the radial modification coefficient of internal gear must be increased, and enlarge the meshing angle a'. This can effectively avoid trochoidal interference or decrease the coefficient of addendum.

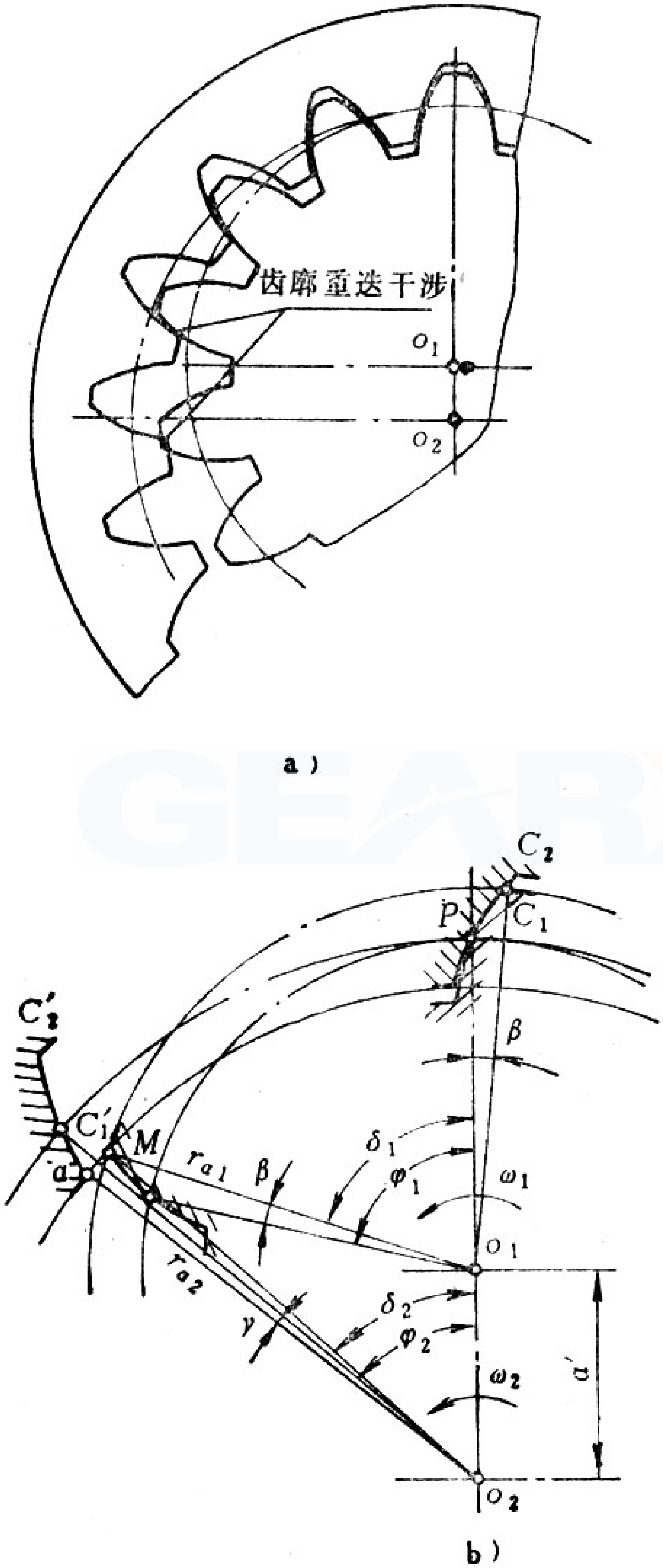

齿廓重迭干涉
o_1
o_2
a）
C_2
C_1
P
C_2'
C_1'
M
a
r_{a1}
r_{a2}
β
δ_1
φ_1
ω_1
o_1
δ_2
φ_2
ω_2
γ
a'
o_2
b）

图 6-51

With regard to trochoidal interference, some references called it addendum angle bump interference, addendum interference, trochoidal interference and second kind interference. This book suggests using trochoidal interference of tooth profile.

Axial assemble interference of internal gear pair

When trochoidal interference occurs, two gears of internal gear pair cannot be installed axially. This kind of addendum collision happening when axial assemble installation carried out is called axial assemble interference.

Radial interference (or trimming)

In regard to two matched gears with less tooth number difference, when axially installing the pinion from the center point of internal gear to the engagement position (as we can see from Figure 6-52), addendum collision occurs and the pinion cannot in the meshing position, this phenomenon is called axial interference. If the pinion is gear shaper cutter, topping of radial feed will occur in the end.

To avoid this, we can take some measures, such as increase the pressure angle, decrease the addendum coefficient, enlarge the tooth number difference z_2-z_1 and increase the axial modification factor of internal gear, etc.

Details please see “topping of radial feed”

Topping of radial feed

When cutting internal gear with gear shaper cutter (Figure 6-52), if the distance (y) between the addendum and center line of gear shaper cutter is larger than the distance (x) between addendum to center line of the manufactured internal gear, collision will occur between the tool addendum and the addendum of the manufactured internal gear. Redundant metal would be cut on the addendum of gear. This is what we called topping of radial feed. The condition to avoid topping of radial feed is $(x\text{-}y)_{min} \geq 0$, with which the formula for avoiding topping of radial feed will be figured out.

In order to avoid topping of radial feed, lots of measures can be taken, such as increase the tooth form angle, decrease addendum coefficient, increase modification factor (including radial or tangent) and enlarge the tooth number difference (z-z_o). What we need to pay attention to is that the smaller z-z_o the value is, the more likely the topping of radial feed would occur. It is not affected by the absolute value of tooth number. Therefore, when the value of z-z_o is relatively small, we need to check whether it has the tendency to have topping of radial feed.

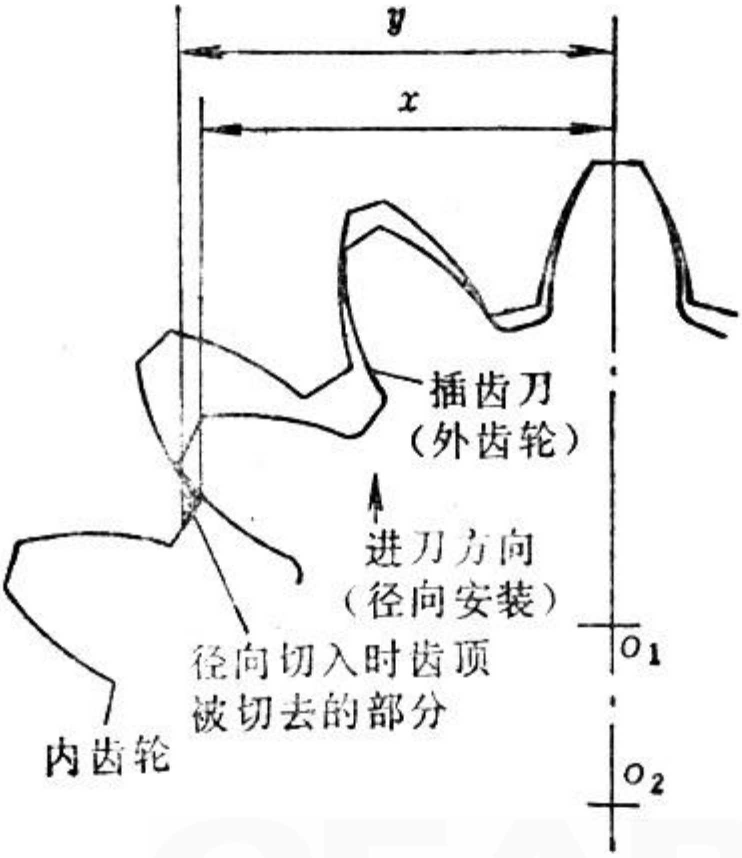


图 6-52

Radial assemble interference of internal gear pair

Under the circumstance of radial interference of internal gear pair, radial assembles installation cannot be realized. Addendum collision occurs during the radial installation process is called axial assemble interference. The essence is radial interference.

Topping in cutting internal gear with shaper cutter

When cutting internal gear with shaper cutter, as the topping is caused by that the addendum circle of gear going beyond the meshing point on the base circle of cutter, then this is called generating topping; if the topping is caused by that the addendum of tool meets the addendum of gear, then this is called topping of radial feed. These are the two topping phenomenon that may occur when processing internal gear with shaper cutter.