8 Gear pair train

8.1 Common terms in gear pair train

Gear train

Optional combination of many gear pair is called gear train. The functions are: speed reduction, speedup, speed change, velocity decomposition, speed composition, shunt transmission, large transmission ratio and large center distance realization. As there are many functions in gear train, it plays a crucial role in mechanical drive and widely applied in many occasion.

Gear train can be classified into gear train with fixed stationary axle and planetary pinion (Figure 8-1).

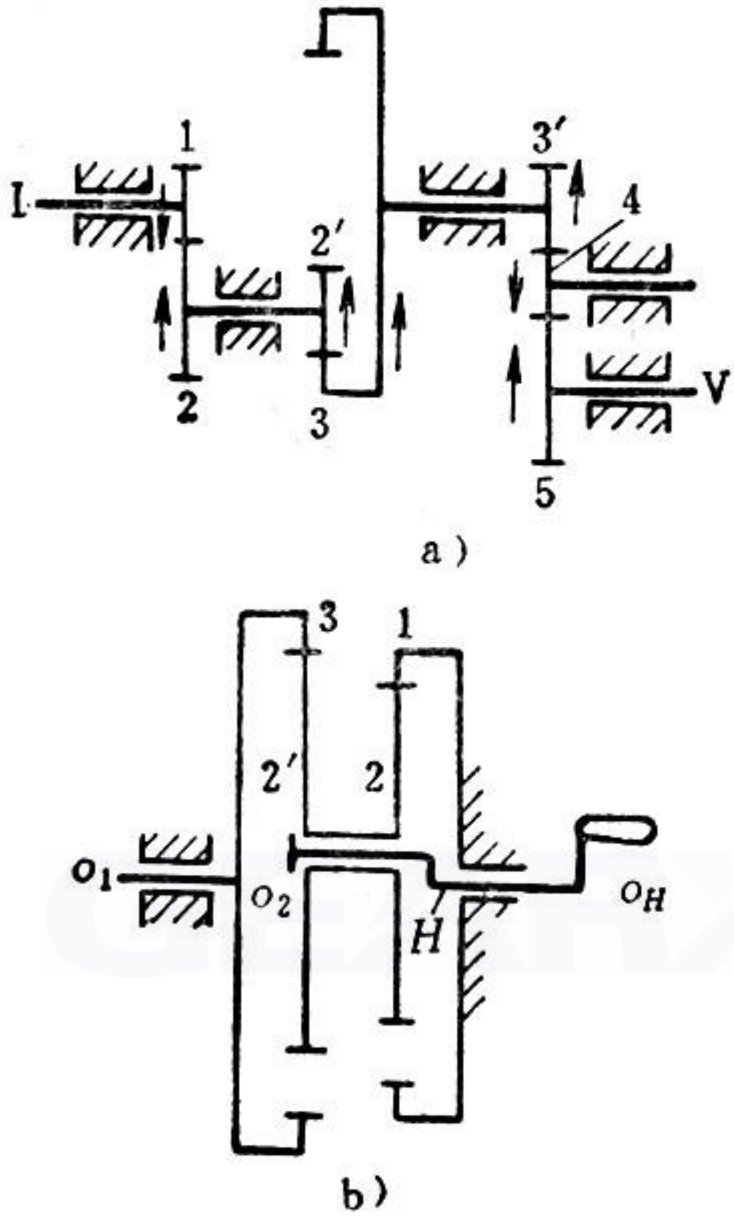


图 8-1

a）固定轴齿轮系　b）行星齿轮系

Gear train of plane fixed axle

Gear train of plane fixed axle is composed of plane gear pair. The features are: the axial line of each gear is parallel; the turning relation of driving axle and driven axle can be shown by $(-1)^m$. M means the number of gear pair in gear train.

Gear train of space fixed axle

There is at least one pair of space gear pair in the fixed axle gear train. The main characteristics of this kind of gear train are: the relation between the driving axle and the driven axle cannot be shown with $(-1)^m$ (m refers to the number of external gear pair). It must be marked with vector symbol on the figure of motion.

Speed reducing gear train

If the angular velocity of the driven gear at end is slower than that of the driving gear at the starting point, then this kind of gear train is called speed reducing gear train.

Speed increasing gear train

If the angular velocity of the driven gear at end is faster than that of the driving gear at the starting point, then this kind of gear train is called speed increasing gear train.

Gear train with fixed axle

If the position of axial line of all the gears in the gear train stays unchanged, then this kind of gear train is called gear train with fixed axle. If all the gear pair in gear train with fixed axle is plane gear pair, then it is called plane gear train with fixed axle; is there are at least one space gear pair in gear train with fixed axle, then it is called gear train with space fixed axle. The main functions are: realizing large transmission ratio, large center to center distance, speed change, direction change, shunt transmission and multi-composition driving, etc.

Transmission ratio of fixed axle gear train

In the working process, the ratio of angular velocity of the two gears or the ratio of speed is called transmission ratio, it can be shown like:

$$i_{1k} = \frac{\omega_1}{\omega_k} = \frac{n_1}{n_k}$$

If the transmission ratio value of each pair of gear pair is already known, then the transmission ratio of gear train with fixed axle is:

$$i_{1k} = i_{12} i_{34} i_{56} \cdots i_{(k-1)k}$$

If the tooth number of each gear in gear train is known, then the value of transmission ratio of gear train with fixed axle can be calculated by the following formula:

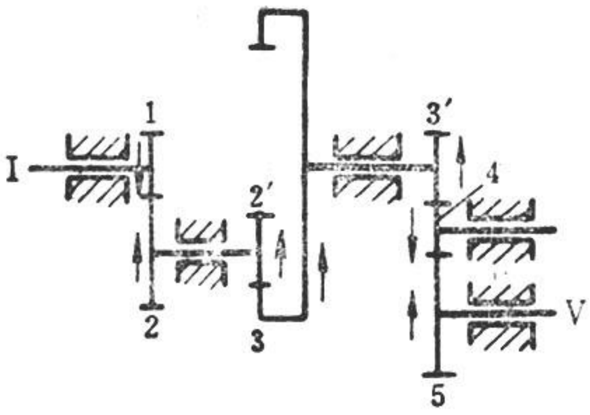
$$i_{1k} = \frac{\text{The product of all the teeth number of driven gear}}{\text{The product of all the teeth number of driving gear}}$$

That is, the product of all driven tooth number divided by that of all driving tooth number.

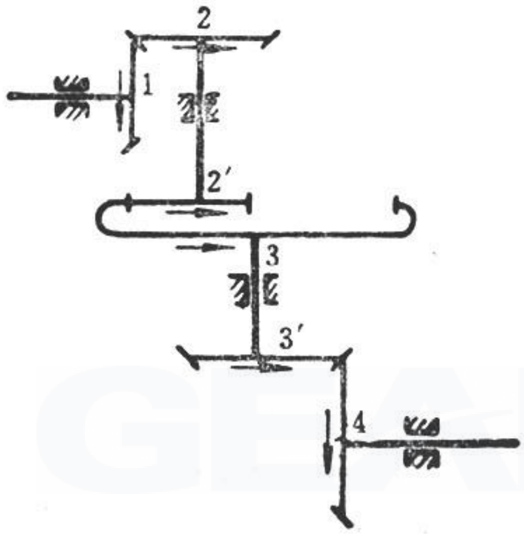
In order to signify the turning relation of the two gears in gear train with fixed axle, there are two methods that can be considered:

(1)For the gear train with fixed axle whose axial line of each gears is parallel to each other, “+” “-” should be added before the formula. When the turning direction of two gears is same, “+” will be used, conversely, “-” will be used.

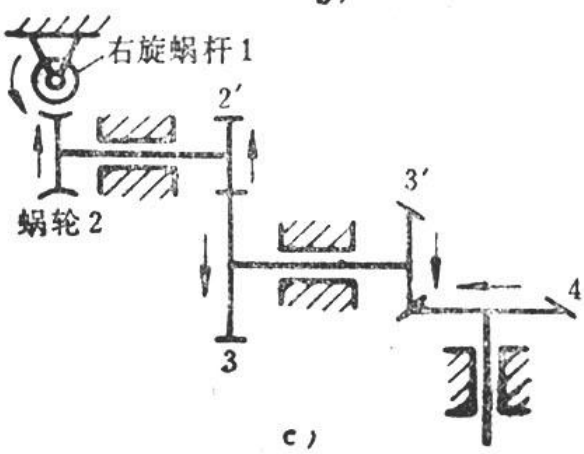
To conform the symbol “+” and “-” of the transmission ratio, two kinds of situation need to be considered: 1) for gear train of plane fixed axle, the symbol is determined by $(-1)^m$, m, here, refers to the number of gear pair in the gear train (Figure 8-2a). 2) For gear train of space fixed axle, the symbol of transmission ratio cannot be determined by the formula of $(-1)^m$. Rather, it is determined by the vector direction marked on the motion figure to figure out the change direction of the two gears (Figure 8-2 b, c). Gear train of plane fixed axle, of cause, can also be confirmed by velocity vector marked method.



a）



b）



c）

图 8-2

a）平面固定轴齿轮系 传动比符号（－1）^{3}为“－”；用矢量表示 1、5 相反 b）空间固定轴齿轮系矢量符号确定传动比为“＋” c）空间固定轴齿轮系用矢量符号表示转向，传动比不计符号

(2) For gear train of space fixed axle with unparallel axial line, there is no “+” or “-” symbol before the value of transmission ratio. But the velocity vector must be marked on its motion figure (Figure 8-2 c).

Transmission efficiency of fixed axle gear train

The effective utilization coefficient of gear train with fixed axle for input power is called transmission efficiency. Transmission efficiency mainly refers to the meshing efficiency of gear pair. The calculation of meshing efficiency is subject to the series connection or parallel connection between their gear pair. If each of the meshing efficiency of gear pair has been known, the method to calculate efficiency with series connection or parallel connection can be used to find the total efficiency. If other conditions are known, then the transmission efficiency can be calculated with meshing power method, torque method and transmission ratio method based on different situation.

Planetary gear train, Epicyclic gear train

When gear train is working, there is at least one axial line of a gear rotating surrounding the fixed axle of another gear. This kind of gear train is called planetary gear train, which was once called epicyclic gear train.

Planetary gear train is composed of planetary gear, planetary carrier, sun gear and engine frame.

According to the meshing characteristics and the shape of tooth profile of gear pair, the main working characters of planetary gear train are: it change the gear train with fixed axle into gear train with dynamic axes; it takes advantage of power and share the weight with many planetary gear train; it uses internal gear pair system reasonably; it has many merits, such as small volume, light weight, large bearing capacity, long service life, high transmission efficiency, smooth working, small noise and large covering scale of transmission ratio. But the transmission efficiency of some planetary gear train will decrease when the transmission ratio increases. Some may even lock by itself. Planetary gear train speed reducer with large power has complicated structure, difficult design process and relatively poor workmanship.

Planetary gear train cannot only fulfill the most functions of gear train with fixed axle, but also can realize speed composition and speed decomposition.

Plane planetary gear train

Plane planetary gear train refers to planetary gear train composed of plane gear pairs. The feature is that all the axial lines of gear are parallel. It is the most widely used planetary gear train.

Space planetary gear train

There is at least one pair of space gear pair in the planetary gear train. The characteristics are: the axial line of each gear is not completely parallel and the corresponding turning relation needs to be shown by the symbol of velocity vector. This kind of planetary gear train is relatively rarely used.

Basic structural unit of planetary gear train

In planetary gear train, structural unit whose axial line is overlap with the fixed axial line and bearing turning moment is called basic structural unit. Various planetary gear trains are always named after basic structural unit. In 2K-H gear train, for example, two central gears and one planetary carrier is basic structural unit; in 3K gear train, three central gears are the basic structural unit.

Centre gear

In planetary gear train, gears meshing with planetary gear train and possessing fixed axial line position is called centre gear (Figure 8-3a, b). It is the basic structural unit of planetary gear train. When external gear serves as central gear, it is called sun gear; when internal gear serves as central gear, it is called internal gear ring.

Sun gear

In planetary gear train, external gear serving as central gear is called sun gear. See Figure 8-3.

Ring gear, annulus

In planetary gear train, external gear serving as central gear is called sun gear. See Figure 8-3.

Planetary gear

In planetary gear train, gear that has dynamic axial line is called planetary gear. The difference between planetary gear train and gear train with fixed axle is whether there is planetary gear. See Figure 8-3

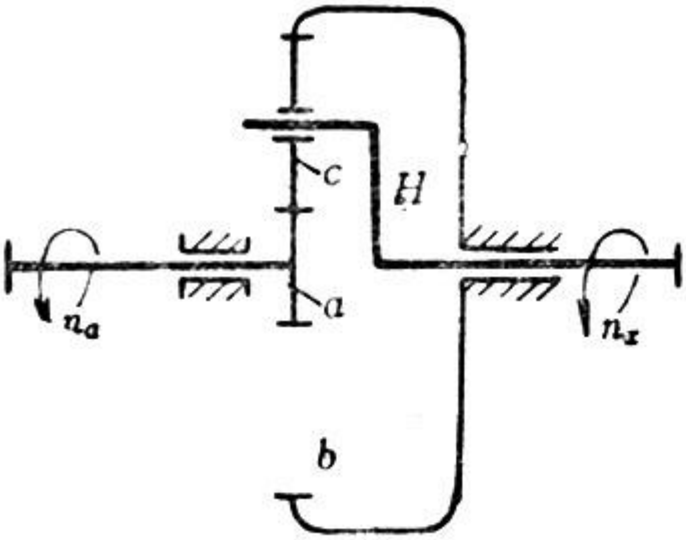


图 8-3

a —中心轮（太阳齿轮） b —中心轮（内齿圈）

c —行星齿轮 H —行星架

Common planetary gear

One planetary gear engages with two central gears at the same time and form two pairs of gear pair. This kind of planetary gear is called common planetary gear, which is shown in planetary gear train with *G*. It is idle gear in its transition mechanism. See c gear in the Figure 8-3.

Planet carrier

In planetary gear train, structural unit bearing the axle of planetary gear and rotating across the axial line of central gear is called planet carrier. Most of them are fundamental structural unit in planetary gear train. The structure shape of planet carrier has various forms. It can be a boom, or a solid of rotation, but the most commonly seen is gear plate. See Figure 8-3

Idle gear

In the moving system of gear train, gear being not only driving gear but also driven gear is called idle gear. Idle gear just affects the symbol of transmission ratio in driving system (relative steering in gear). The size of total center distance not affects the value of transmission ratio.

Single planetary gear train

It refers to planetary gear train that has only one planet carrier. Its planetary gear can be single row or double row. There would be 1, 2 or 3 central gears. Single planetary gear train is the basic type of planetary gear train. It is the basic unit that can be used to analyze the motion, force as well as doing design calculation.

Simple planetary gear train

It refers to single planetary gear train that has only one planet carrier and the degree of freedom is 1.Obviously, there is only one central gear as its active structural unit for 2K-H gear train. This kind of planetary gear train is the most widely used type.

Positive mechanism

2K-H gear train with two sun gears and one planet carrier or with two ring gears and one planet carrier, its transmission ratio of transforming mechanism is positive, and then the transforming mechanism of this kind of 2K-H gear train is called positive mechanism. The structural characters: two central gears or two sun gears or two ring gears. It is not superior to negative mechanism and it mainly shows in: as the transmission ratio increase, the transmission efficiency become lower and self-lock problem may even occur sometimes.

Negative mechanism

The transmission ratio of transforming mechanism of 2K-H gear train with sun gear, ring gear and planet carrier as structural unit is negative, then the transforming mechanism of 2K-H gear train is called negative mechanism. Its structural characteristic is: the two central gear is external gear (sun gear) and internal gear (ring gear) respectively. The main working characters: high transmission efficiency, large bearing capacity, large scale of transmission power, simple structure, small volume and good performance. It is the most widely used planetary gear train.

Conversion mechanism

See “planet carrier fixed method”.

Multiple-stage planetary gear train

It is a combined planetary gear train composed of two or more same-type single planetary gear train. When conducting transmission ratio calculation, motion analysis, force analysis and design calculation, multiple-stage planetary train need to be decomposed into single planetary gear train. See Figure 8-4.

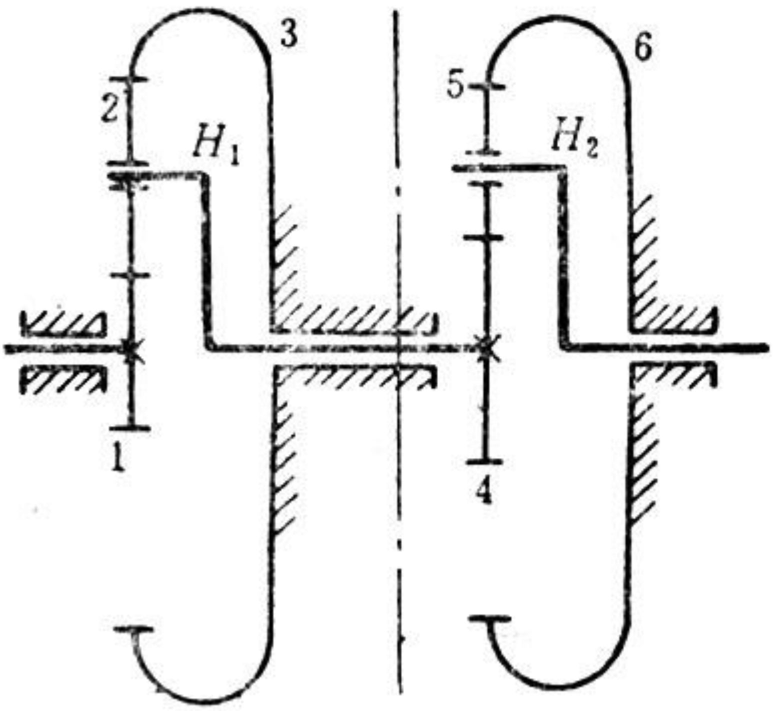


图 8-4

Compound planetary gear train

Gear train with fixed axle and planetary gear train, or more than two different types of single planetary gear train, when composing together, become compound planetary gear train. Compound planetary gear train can realize many functions that cannot be realized through gear train with fixed axle, single planetary gear train, simple planetary gear train and differential gear train. For example, large scale of transmission ratio can be realized; various speed diversions, speed composition can also be fulfilled; it can also help change the degree of freedom of gear train. Generally, compound planetary gear train has complicated structure, relatively poor workmanship and high cost.

Differential gear

It refers to single planetary gear train that has only one planet carrier and the degree of freedom is 2.It is also refers to single planetary gear train whose three basic structural unit are all active structural units. Generally, two central gears are all driven by two electric motors. Differential gear can be designedly conveniently into gear mechanism that has speed decomposition or composition of velocities.

Figure 8-5 is NGW typed differential gear system.

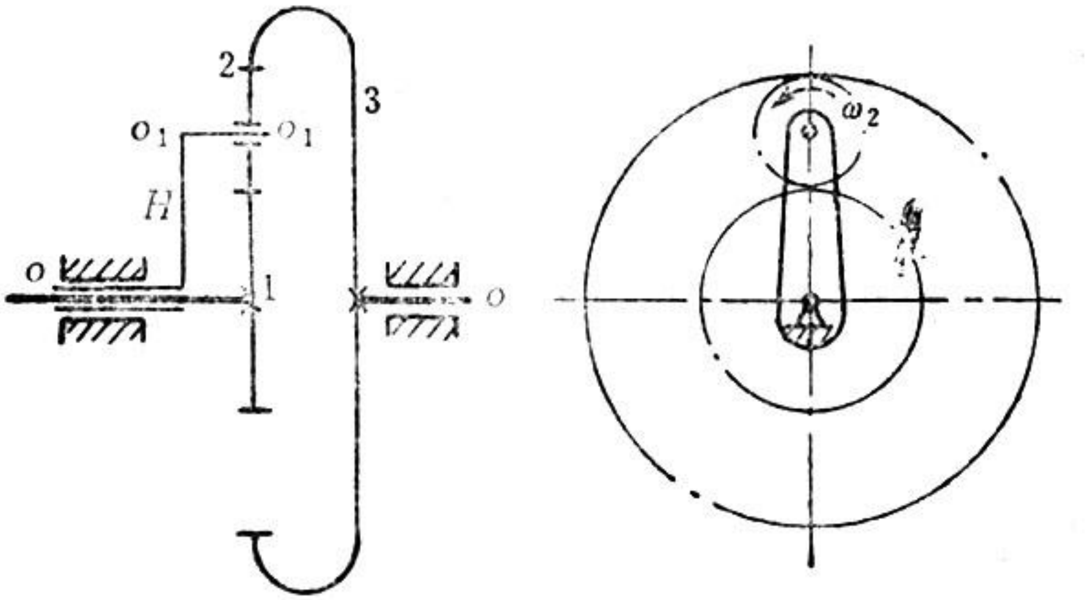


图 8-5

Enclosed differential gear train

Enclosed differential gear train is a special compound gear train. In this compound gear train, the planetary gear train part is differential gear whose degree of freedom is 2, the other gear train with fixed axle or planetary gear train connect the two basic structural units of differential gear (enclosed) and make the degree of freedom of compound gear train lower to 1.This kind of compound gear train is called enclosed differential gear train (Figure 8-6).

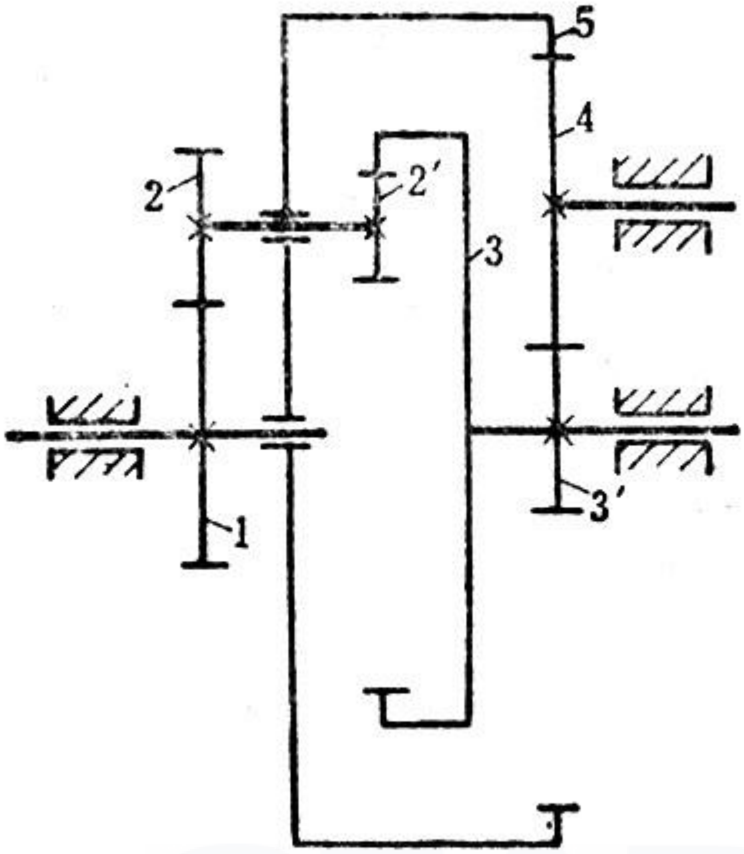


图 8-6

Resolution of velocity of differential gear train

Differential gear train has three basic structural units and two degree of freedom. The speed of the basic structural unit of a driving gear can be deposed into different speed of another two basic structural units based on required proportion relation in certain conditions. This kind of motion relation is called the resolution of velocity of differential gear train.

A typical example is ZUWGW gear system. Taking the differential gear case of rear axle of motor vehicle, when z_a—z_b,

$$i_{ab}^{H} = \frac{\omega_a - \omega_H}{\omega_b - \omega_H} = \frac{z_b}{z_a} = -1$$

$$\omega_a - \omega_H = \omega_H - \omega_b \quad 2\omega_H = \omega_a + \omega_b$$

Together with auxiliary condition of the motor bend,

$$\frac{\omega_a}{\omega_b} = \frac{r - L}{r + L}$$

Then one can know that when $r = \infty$ (driving in a straight way), the speed of three basic structural units is equal $n_a = n_b = n_H$ .When $r \neq \infty$ (driving in a bend way): please see Figure 8-7.

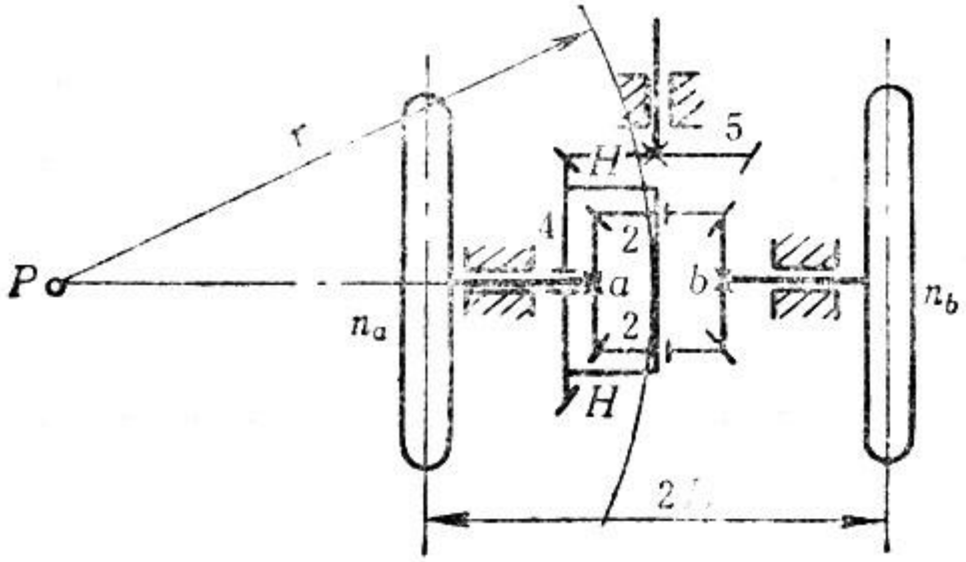


图 8-7

$$\begin{cases} n_a = (r - L)n_H / r \\ n_b = (r + L)n_H / r \end{cases}$$

Composition of velocity of differential gear train

Differential gear train has two degree of freedoms. Therefore, if one wants to know the motion law of the third basic structural unit, he must learn the independent parameters of the other two basic structural units. This kind of speed relation is called the composition of velocities of differential gear train.

Such as ZUWGW, $\omega_b i_{ab}^H - \omega_a = (i_{ab}^H - 1) \times \omega_H$ when $z_b = z_a$, $\omega_a + \omega_b = 2\omega_H$. This can explain that when any speed value of two basic structural units are known, the speed of the third basic structural unit can be gotten.

Select condition of the number of each gear teeth in planetary gear train

Planetary gear train is a gear system where many gears are engaged and their mutual relation is strictly formed. Therefore, the tooth selection of each gear should meet the following requirements: to make sure a certain transmission ratio; to ensure the same axial line condition of central gear and planet carrier (concentric condition) and satisfy technology requirement on installation; to ensure there is no collision between each planetary gear (consecutive condition).

Concentric condition

When designing planetary gear train, the basic structural unit of gear train rotating surrounding the same axial line should be ensured. Then the limit condition in selecting the tooth number is called concentric condition. That means the center distance between planetary gear and the two gear pairs must equal, which means $r_a' + r_c' = r_b' \pm r_c'$ ("+" is used in external gearing while "-" is used in internal gearing) (Figure 8-8).

1.2 K-H (NGW) gear train

For NGW gear pair composed of standard gear pair and gear pair with reference center distance, its concentric condition is:

$$\begin{cases} z_c = (z_b - z_a)/2 \\ z_c = (i_{ab}^H - 2)/2 \end{cases}$$

One can learn from the formula that the tooth number of the two central gears should be odd number or even number at the same time.

Or $$\begin{cases} z_c = z_a i_{aH}^b / 2 - z_a \\ z_c = z_\Sigma - z_a \\ z_\Sigma = z_c + z_a \end{cases}$$

From the formula, one can know that $z_a i_{aH}^b$ must be an even number to meet the concentric condition.

When it is gear pair with modified center distance,

$$z_a + z_c / \cos\alpha'_{ac} = (z_b - z_c)/\cos\alpha'_{bc}$$

The selection of its tooth number is very flexible.

2.2K-H(NW、WW、NN) gear train

For planetary gear train gear train composed of standard gear pair and gear pair with reference center distance, the concentric condition is:

$$\begin{cases} \text{NW} \quad z_a + z_c = z_b - z_d \\ \text{WW} \quad z_a + z_c = z_b + z_d \\ \qquad\qquad z_a - z_b = z_d - z_c \\ \text{NN} \quad z_a + z_c = z_b + z_d \\ \qquad\qquad z_a - z_b = z_d - z_c \end{cases}$$

Among these three types of gear train, no matter what kind of gear pair they belong to, the center distance of two gear pair must be equal, that is, $a'_{ac} = a'_{bd}$.

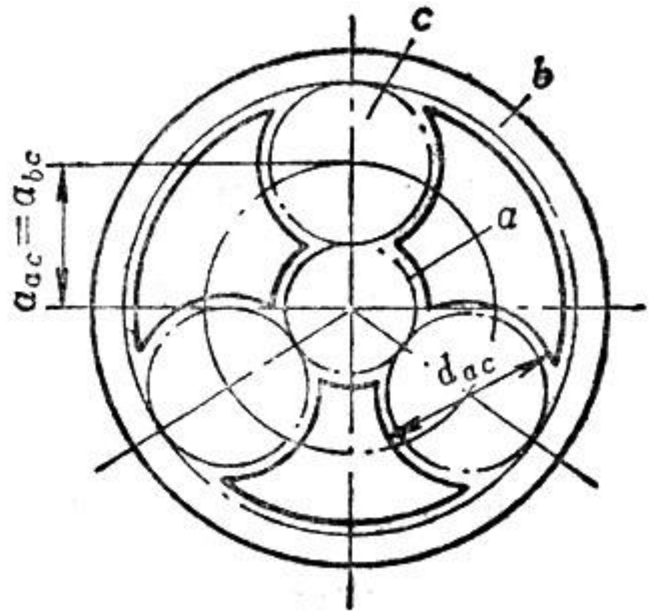


图 8–8

Consecutive condition

After the planetary gear is installed between the central gears, the tip surfaces of the adjacent planetary gear not collide and there are clearance between them, conditions should be satisfied for the above mentioned is called consecutive condition. Then:

$$d_{ac} < 2a_{ac}\sin\frac{\pi}{n_p}$$

Condition of transmission ratio

The first condition that must be satisfied is that planetary gear train must realize the vested transmission ratio. It is the first limitation to determine the tooth number. For single planetary gear train, the formula is workable.

$$i_{ab}^{H} = \frac{\omega_a - \omega_H}{\omega_b - \omega_H} = \frac{n_a - n_H}{n_b - n_H}$$

In $i_{aH}^{b} = \frac{\omega_a - \omega_b}{\omega_H - \omega_b} = 1 - i_{ab}^{H}$, i_{ab}^{H} is the function of gear tooth in each gear pair. If the angular speed (or rotating speed) is already known, tooth number of different planetary gear train can be calculated:

For 2K-H (NGW) gear train:

$$z_b = (i_{aH}^{b} - 1)z_a$$

$$z_\Sigma = z_b + z_a = i_{aH}^{b} / 2$$

$z_b = 2z_\Sigma - z_a$

For 2K-H(NW、WW、NN) gear train:

$i_{aH}^b = 1 \pm z_c z_b / z_a z_d$

In the formula, the symbol “+” is used in NW gear train with negative mechanism while the symbol “-” is used in WW and NN gear train with positive mechanism. The conditional formula of transmission ratio of NW gear train with negative mechanism is: $i_{aH}^b (z_a z_d + z_b z_c) / z_a z_d$.The conditional formula of transmission ratio of NW gear train with positive mechanism is: $i_{Ha}^b = z_c z_d /(z_a z_d + z_b z_c)$.

Erecting condition

When designing planetary gear train, what should be ensured is that each planetary gear train is installed orderly between the two central gears. This requires a reasonable selection of tooth number of z_a, z_c and z_b gears, which is called erecting condition. The condition is mainly showed in NGW gear train. The installation condition is: the sum of tooth number of two central gears over the planetary gear number should be an integer.

$$q = z_a + z_b / n_p = \text{int}\,eger$$

$$or \quad q = 2z_\Sigma / n_p = \text{int}\,eger$$

$$z_\Sigma = z_a + z_c = z_b - z_c$$

2K-H typed gear train

Planetary gear train taking two central gears (K) and a planet carrier (H) as its basic structural units is called 2K-H type gear train. 2K-H type gear train has three types: single, double and multiple kinds. It can be classified into negative mechanism and positive mechanism based on its conversion mechanism. It is the most widely used planetary gear train that has various types.

2K-H type gear train (negative mechanism), which taking sun gear, ring gear and planet carrier as its basic structural unit, has high driving efficiency, large bearing capacity and large scale of transmission power. The structure is also simple but it has good performance. 2K-H type gear train (positive mechanism) taking two ring gears, one planet carrier or two sun gears, one planet carrier as its basic structural units is not as good as the negative mechanism in meshing characteristics and structural features.

NGW type gear train

In planetary gear train, NGW type gear train refers to 2K-H type gear train named after the type of gear pair (symbolically) (Figure 8-9). NGW type gear train refers to single planetary gear train (Figure 8-3) composed of ring gear *b*, sun gear *a*, common planetary gear *c* and planet carrier *H*. It belongs to negative mechanism. It is widely used because of its high transmission efficiency, small volume, light weight, simple structure, good workmanship and large scale of transmission power. Relatively small transmission ratio is the deficiency.

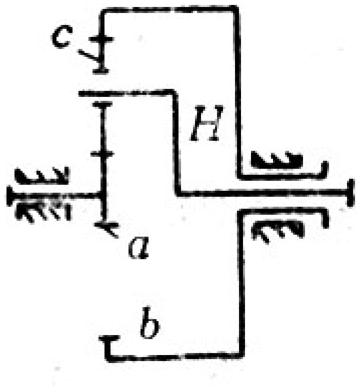


图 8-9

N type gear train

In planetary gear train, N type gear train refers to K-H-V type gear train named after the type of gear pair (symbolically) (Figure 8-10). N type gear train is a single planetary gear train composed of ring gear *b*, planetary gear *c*, planet carrier H and *W* output mechanism. It belongs to positive mechanism. It has large transmission ratio, compact structure, good workmanship and relatively large axial bearing force for the central axle of planetary gear. It is suitable for transmitting equipment with small power and short-term working condition.

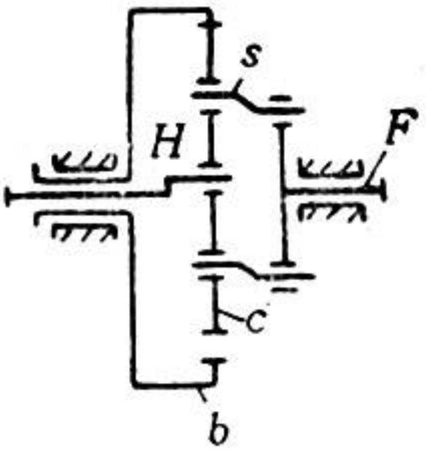


图 8-10

NGWN type gear train

In planetary gear train, NGWN type gear train refers to 3K type gear train named after the type of gear pair (symbolically). It refers to a single planetary gear train (Figure 8-11) composed of two ring gears (N), one double planetary gears (one is common planetary gear G), one planetary carrier without bearing torque and one sun gear (W). It belongs to positive, negative mechanism. It has compact structure, small volume, light weight and large transmission ratio, but the workmanship is relatively poor and its transmission efficiency is lower than NGW type gear train. It is suitable for transmitting occasion with small power and short-term working condition.

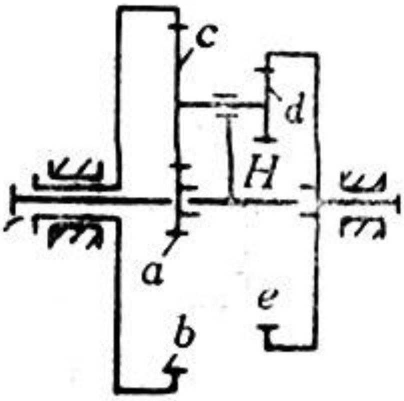


图 8-11

NW type gear train

In planetary gear train, NW type gear train refers to 2K-H type gear train named after the type of gear pair (symbolically) (Figure 8-12). NW type gear train refers to single planetary gear train composed of ring gear, sun gear, double planetary gears and planet carrier. It belongs to negative mechanism which has high transmission efficiency and large transmission ratio. The radial size is smaller than NGW type gear train but it has large axial size than NGW type gear train. The workmanship of double planetary gear is relatively poor.

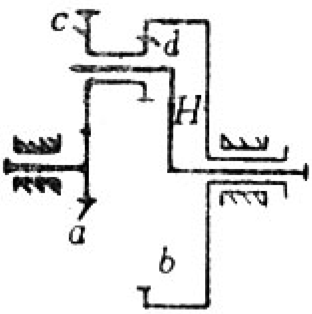


图 8-12

NN type gear train

In planetary gear train, NW type gear train refers to 2K-H type gear train named after the type of gear pair (symbolically) (Figure 8-13). NN type gear train refers to single planetary gear train composed of two ring gears (*N*), one double planetary gears and one planet carrier. It belongs to positive mechanism. It has large transmission ratio but relatively low transmission efficiency, which decrease when the transmission ratio increases. When taking planet carrier as its driven unit, self-lock condition may occur. Besides, it has relatively large vibration noise. This kind of planet carrier is mostly used in short-term of intermittent working occasions.

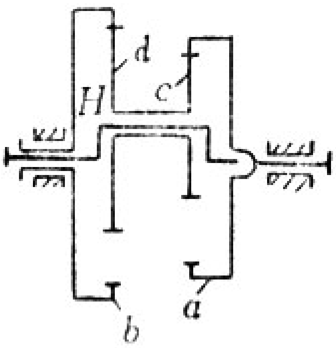


图 8-13

WW type gear train

In planetary gear train, WW type gear train refers to 2K-H type gear train named after the type of gear pair (symbolically) (Figure 8-14).WW type gear train refers to single planetary gear train composed of two sun gears, one double planetary gears and one planet carrier. It belongs to positive mechanism. It has large transmission ratio but relatively low transmission efficiency, which decreases as the transmission ratio increases; with large profile size and heavy weight, the gear train is not used in power driving and dividing transmission in general.

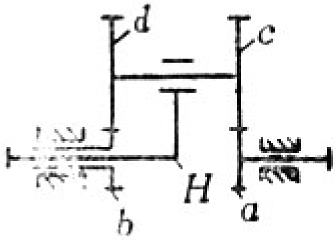


图 8-14

ZUWGW type gear train

In planetary gear train, NUWGW type gear train refers to 2K-H type gear train named after the type of gear pair (symbolically) (Figure 8-15). ZUWGW type gear train is planetary gear train composed of one common planetary gear and one planet carrier and it takes two bevel gears as its central gear. It belongs to negative mechanism. With relatively high transmission efficiency, it is mainly used in differential gear train.

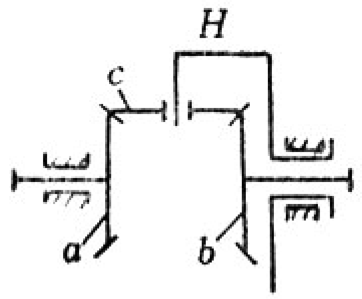


图 8-15

Double NGW type gear train

Double NGW type gear train is composed of two NGW type single gear trains which are in series connection (Figure 8-16).This kind of planetary gear train not only overcomes the deficiency of low transmission ratio of NGW gear ratio but also retains the merits of NGW type gear train. In Figure 8-16, a planet carrier is the moving link; Figure 8-16b shows high speed fixed planet carrier. High velocity driving is adoptable at this time.

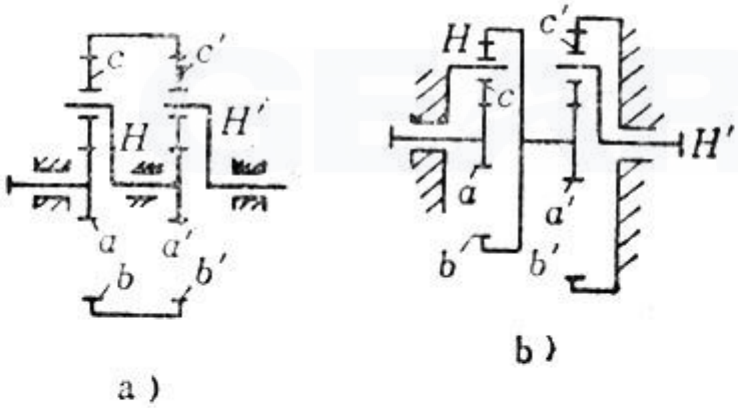


图 8-16

3K type gear train

3K type gear train refers to planetary gear train (Figure 8-17) taking three central gears as basic structural unit. The main characteristics are: three central gears are used to transmit turning moment. Without bearing turning moment, planet carrier is only served to support planet gear axle. This kind of planetary gear train has compact structure, small volume and large transmission ratio but with low transmission efficiency and poor workmanship. It is suitable for working in occasion with small-and-medium power and short-term working condition.

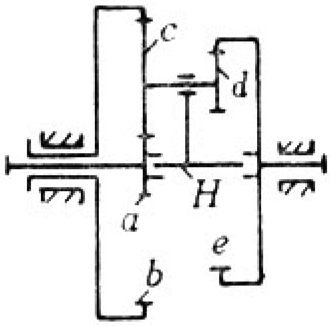


图 8-17

K-H-V type gear train

K-H-V type gear train refers to planetary gear train (Figure 8-18) taking one central gear *K*, one planet carrier *H* and one output axle *V* as basic structural unit. It has only one central gear. The driving force in complicated motion of planetary gear outputs by *V* shaft through the output mechanism of equal angular velocity ratio. As the planetary gear shaft and output shaft are interlacing, the force must be outputted by the above mentioned two sides towards coupling (*W* mechanism). The design of tooth profile, output mechanism and workmanship are all highly required in this kind of planetary gear train.

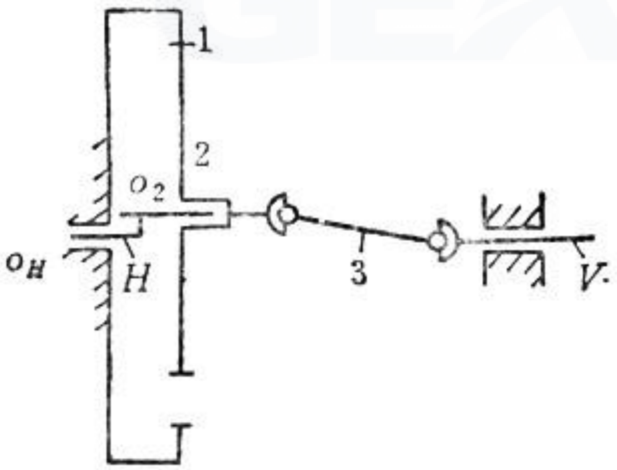


图 8-18

Transmission ratio of planetary gear train

Transmission ratio of planetary gear train refers to the ratio of angular velocity or turning moment of any two basic moving structural units. It also refers to the ratio of angular velocity or turning moment of driving or driven basic structural unit of planetary gear train.

There are lots of methods in planetary gear train calculation. The mainly are:

fixed planet carrier method, analyzing method in speed diagram and vector diagram analyzing method, etc.

Fixed method of planet carrier

The main distinction of planetary gear train and gear train with fixed axle is the turning motion of planet carrier, which leads the axial line of planetary gear rotating across that of central gear. If the planet carrier can be fixed accordingly, the planetary gear train may become gear train with fixed axle. At that time, the speed and transmission ratio can be calculated according to the method that is used in gear train with fixed axle calculation.

The accordance of fixed method of planet carrier is: when adding the same moving speed on each structural unit of the whole mechanism, the correspondent moving relation between each unit would not be affected. Based on the fundamental principle, one can imagine adding $(-\omega_H)$ angular speed at every unit in the planetary gear train, then the angular speed of each units corresponding to planet carrier is $\omega_H^H = \omega_H - \omega_H = 0$ $\omega_a^H = \omega_a - \omega_H$ $\omega_b^H = \omega_b - \omega_H$ $\omega_c^H = \omega_c - \omega_H$ respectively. At this time, planet carrier is in relative rest and the planetary gear train becomes an imaginary gear train with fixed axle. And that is what we called conversion mechanism of planetary gear train. Obviously, the transmission ratio of imaginary gear train with fixed axle is:

$$i_{ab}^H = \frac{\omega_a^H}{\omega_b^H} = \frac{\omega_a - \omega_H}{\omega_b - \omega_H}$$

$\text{In} = (\pm)\dfrac{\text{The product of tooth number of each driven gear}}{\text{The product of tooth number of each driving gear}}$, "±" is determined by the turning direction relation between the driving gear and the driven gear.

For differential gear train, central gear a、b are all active moving units. Then:

$$\omega_a = \omega_b i_{ab}^H - (i_{ab}^H - 1)\omega_H$$

There are two degree of freedom in differential gear train, therefore, the angular velocity (or turning moment) of two basic structural units in the three basic structural units must be given to confirm the third one. When all the three angular velocity are gotten, the transmission ratio of any two basic structural units can be obtained.

For simple planetary gear train, the degree of freedom is 1 and only two out of three are moving links. For example, 2K-H type gear pair has one central gear fixed (such as $\omega_b = 0$), then the transmission ratio between the central gear a and planet carrier is:

$$i_{aH} = \frac{\omega_a}{\omega_H} = -(i_{ab}^H - 1) = (1 - i_{ab}^H)$$

When calculating transmission ratio through fixed method of planet carrier, one need to note that: the method is only adoptable in plane (or space) planetary gear train whose turning axial lines of three basic structural units are mutually parallel; in that formula, i_{ab}^{H} is the transmission ratio of an imaginary conversion mechanism, one need to pay attention to the confirmation of symbol of transmission ratio "±" and the turning relation of ω_a, ω_b, ω_H. If any ω is set to be positive (or negative), symbol of the other two ω can be got. Then, by adding the symbol "+" and "-" into the formula, one can get the exact transmission ratio. For planetary gear train or non-2K-H type gear train, one need to separate it into single 2K-H type gear train and gear train with fixed axle and link each of the listed calculating formula to get the final result. There are various planetary gear train and compound planetary gear train. So it is unpractical to give a general idea to solve different problems. Specific issue needs special analysis.

Using fixed method of planet carrier to get transmission ratio has many merits, such as clear conception and exact calculation. But it has limitation in some space system train.

Transmission ratio calculation of compound planetary gear train

Transmission ratio of compound planetary gear train refers to the rate of angular speed (or turning moment) of input axle and output axle. The calculation method combines the calculation method of gear train with fixed axle and single planetary gear train. That means one need to separate the compound planetary gear train into many single planetary gear trains or single planetary gear train and gear train with fixed axle first, then list the corresponding calculation formula of transmission ratio based on the calculation method of single planetary gear train or gear train with fixed axle, analyzing the connection between each calculation method and find out the required unknown quantity, finally, with these formulas in hand, one can get the unknown factor.

The calculation of transmission ratio of compound planetary gear train must be paid attention in the following factors:

(1) The resolution of compound planetary gear train

The resolution of compound planetary gear train means to decompose compound planetary gear train into single planetary gear train and gear train with fixed axle (maybe without gear train with fixed axle). According to the definition of single planetary gear train, firstly, one need to find out all the planetary gears in compound planetary gear train, then one needs to find out the planet carrier and central gear. Finally, single planetary gear train will come out. Besides, one need to note that if there is a planet carrier, there is a single planetary gear train. The number of single planetary gear train included in compound planetary gear train is equal to the number of planet carrier. Single planetary gear train mainly refers to 2K-H, 3K, and K-H-V type gear train. When all the single planetary gear train is in decomposition, the rest gear pair would certainly compose one or more gear train with fixed axle.

(2) When it comes to 3K gear train, one need to follow the specific solution to list the formula of transmission ratio.

(3) When using fixed method planet carrier to get transmission ratio and using the universal formula of conversion mechanism, one need to pay close attention to the symbol of i_{ab}^{H} and the symbol of ω_a, ω_b and ω_H.

(4) When listing the simultaneous equations, one must find the connection between each formula and list a simple simultaneous equations with which one can easily get the answer.

Calculate transmission ratio with vectorgram analysis method

It is a method used to calculate transmission ratio based on kinematic principle of planetary gear train. This method can help show the nature of speed vector crossing its axis and get the velocity vector with vector addition, and then transmission ratio can be calculated. The speculation is that the direction of axis (relative speed vector is the surrounding instantaneous axis) crossed by speed vector is determined in accordance with right hand rule. After the scale being chosen and the speed vector formula being listed, the unknown value of speed can be calculated, so does the transmission ratio. See Figure 8-19.

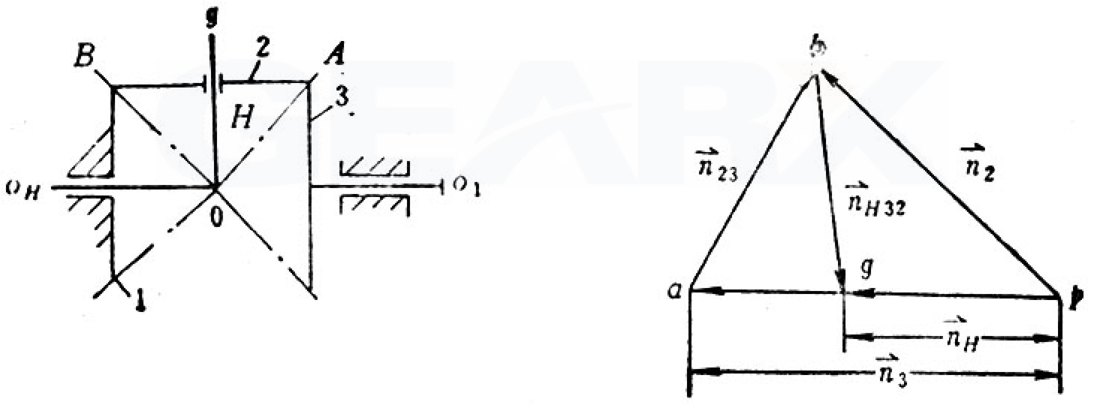


图 8-19

This method is quite suitable to calculate the transmission ratio of planetary gear train including bevel gear pair.

Calculate transmission ratio with velocity plan analysis method

It is a method used to calculate transmission ratio based on the kinematic principle of planetary gear train. The method can help calculate transmission ratio when speed and angular speed are worked out in accordance with velocity diagram drawn based on instantaneous center characteristics. In any two instrument units there is an instantaneous center in planetary gear train. The instantaneous center of moving link and machine frame is called primary center, at where the speed is zero. When the moving link rotates across the primary centro, the speed at every point of the unit is presented in the shape of a triangle. The instantaneous center formed by the two moving links is called secondary centro, at where the absolute velocity of two units is equal and their relative velocity is zero. Obviously, the intersecting point of the axis of central gear and planet carrier and the transverse plane of gear is the primary centro, while the pitch point of two engaged gears is the secondary centro (if one gear is a fixed unit, then it is primary centro). The instantaneous center of two indirect connected units can be calculated by “the theorem of three centers”.

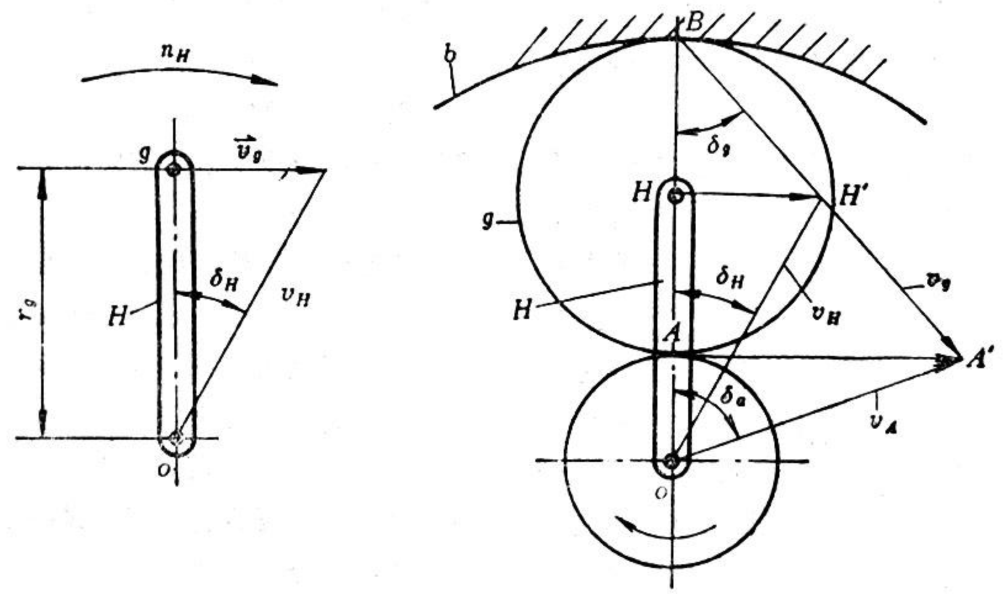


图 8-20

When solving transmission ratio with velocity plan analysis method, one should firstly analyze its motion characteristics to get each transient center, choose the scale of velocity and length and draw the velocity plan in accordance with the known conditions. Then the velocity of unknown point and the distance from the point to transient center can be calculated. Then, making the two numbers times scale respectively and calculate the actual velocity and distance, the ratio between the two is called angular velocity ω, with which one can get the angular velocity value of central gear and planet carrier and finally get the transmission ratio based on the speed direction presented in the velocity ratio.

Using velocity plan analysis method to calculate transmission is very vivid and convenient. But it can only applied in plane planetary gear train. It is quite sophisticated to calculate the relation between transmission ratio and tooth number with this method.

Calculate transmission ratio with torque method

It refers to the method used to calculate the transmission ratio of planetary gear train in accordance with the basic principle of inverse ratio between velocity and torque without considering friction. For example: $i_{ab}=\frac{\omega_a}{\omega_b}=-\frac{T_b}{T_a}$, “-” means the direction of T_a、T_b is opposite. Choose the scale, write the vector formula and draw the vector diagram in accordance with the torque beard by the three basic structural unit (the direction should follow right hand rule) and vector addition. Then, one should measure the unknown torque and divided the scale and get the actual torque. Then the transmission ratio can be gotten.

This method cannot only help calculate the transmission ratio but also help figure out the torque, laying the foundation for strength calculation and transmission efficiency calculation. But the calculation is complicated. It is applicable to the designing process.

Basic dynamic capacity

For planetary gear train, when the sun gear serves as driving unit, it works in 1 million rmp. The definition: when the reliability is 90 percent, the input torque is the basic dynamic capacity of the planetary gear train.

Transmission efficiency of planetary gear train

The effective utilization factor η to input power of planetary gear train is called transmission efficiency. It refers to the ratio between the output power (productive resistance power) and input power (driving power). It is a number less than 1 and generally presented in percentage. There are various planetary gear trains and their driving units and driven links are of different types. Therefore the range of variation is quite big. So it is necessary to set its transmission efficiency. Roughly, there are two ways to set transmission efficiency. One is a calculation method and the other is a testing method. As the transmission efficiency is affected by lots of factors (such as processing technology, installation, precision class, material and lubricated condition etc.), the number calculated may have a far cry with the actual result. Practices prove that the calculation of transmission efficiency of planet gear pair under the guidance of testing method is more important than using other method.

The transmission efficiency of planetary gear train generally includes the meshing efficiency, bearing efficiency, hydraulic efficiency, and load sharing efficiency, in which the meshing efficiency is of the greatest importance.

The transmission efficiency of planet gear pair changes as the structure type changes; for the same type, the transmission efficiency reduces as the transmission ratio increases; for the same type, the transmission efficiency changes as the renewal of driving or driven devices; it changes along with the changes of technology, lubricated condition and utilization condition.

There are a great many theoretical methods to calculate the transmission efficiency of planetary gear train, and the main method comes to the engagement power of conversion mechanism and torque method of conversion mechanism and transmission ratio method, etc. Among all the listed, engagement power method of conversion mechanism is the most widely used one.

Engagement power method of conversion mechanism

It is a method using the engagement power of the imaginary conversion mechanism of planetary gear train to obtain the transmission efficiency of planetary gear train. The basis of the method is: comparing conversion mechanism and other mechanism of planetary gear train, their relative movement relation are the same. This means that their engagement power loss is the same as long as the working external torque keeps same. However, as n_i becomes $(n_i - n_H)$, the transmitted power changes on each of the instrument units. $(n_i - n_H)$. It can be positive or negative. That proves that there may be some changes between the direction of torque and speed. But as long as the external torque of planetary gear train and conversion mechanism keep same, their engagement power stays the same. Then:

$$P_a^H = T_a n_a^H = T_a(n_a - n_H)$$

The power relation of planetary gear train and conversion mechanism is:

$$P_a^H / P_a = T_a n_H^H / T_a n_a = (n_a - n_H)/n_a$$

$$= 1 - 1/i_{aH}$$

Two occasions need to be noticed when using the formula:

(1) The sun gear serves as driving unit and the planet carrier is driven units. The input power is $P_a = T_a n_a$. At this time, changes may occur or not occur in the driving and driven relation of sun gear *a*: when $1 - 1/i_{aH} > 0$, sun gear *a* still serve as driving unit, then:

In $\eta_{aH} = [1 - \eta^H(1 - i_{aH})]/i_{aH}$, η^H is the total engagement efficiency of conversion mechanism, which is equal to the product of the engagement efficiency of each gear pair.

When $1 - 1/i_{aH} < 0$, sun gear *a* becomes a driven unit in the conversion mechanism, then:

$$\eta_{aH} = [\eta^H - (1 - i_{aH})]/i_{aH} \cdot \eta^H$$

(2) When the sun gear is the driven unit while the planet carrier is the driving unit, the output power is $P_{a0} = T_a n_a$.

When $1 - 1/i_{aH} > 0$, sun gear *a* is still a driven unit in the conversion mechanism, then:

$$\eta_{aH} = i_{aH}\eta^H / [\eta^H - (1 - i_{aH})]$$

When $1 - 1/i_{aH} < 0$, sun gear *a* becomes driving unit in the conversion mechanism, then:

$$\eta_{aH} = i_{aH} / [1 - \eta^H(1 - i_{aH})]$$

Calculate transmission efficiency with transmission ratio method

It is one way to calculate the transmission efficiency of planetary gear train. This method is suitable to be applied in calculating the transmission efficiency of enclosed differential gear train. The basis of using this method to calculate transmission efficiency is that transmission efficiency can be indicated by the relation between the ratio of torque and the ratio of transmission ratio, which is:

$$\eta = -N_2 / N_1 = T_2\omega_2 / T_1\omega_1 = -\frac{T_2}{T_1} \Big/ \frac{\omega_1}{\omega_2} = \tilde{i} / i_{12}$$

The expression is:

$$i_{1n} = f(i_1、i_2 \cdots i_n)$$

$$\tilde{i}_{1n} = f(\tilde{i}_1、\tilde{i}_2 \cdots \tilde{i}_n)$$

On the basis of T_n / T_1、ω_1 / ω_n, transmission efficiency can be calculated.